AQA

AS
FURTHER MATHEMATICS
7366/1

Paper 1

Mark scheme

June 2019

Version: 1.0 Final

196A7366/1/MS

Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts. Alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Examiner.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this mark scheme are available from aqa.org.uk

Mark scheme instructions to examiners

General

The mark scheme for each question shows:

- the marks available for each part of the question
- the total marks available for the question
- marking instructions that indicate when marks should be awarded or withheld including the principle on which each mark is awarded. Information is included to help the examiner make his or her judgement and to delineate what is creditworthy from that not worthy of credit
- a typical solution. This response is one we expect to see frequently. However credit must be given on the basis of the marking instructions.

If a student uses a method which is not explicitly covered by the marking instructions the same principles of marking should be applied. Credit should be given to any valid methods. Examiners should seek advice from their senior examiner if in any doubt.

Key to mark types

M	mark is for method
dM	mark is dependent on one or more M marks and is for method
R	mark is for reasoning
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
F	follow through from previous incorrect result

Key to mark scheme abbreviations

CAO	correct answer only
CSO	correct solution only
ft	follow through from previous incorrect result
‘their’	indicates that credit can be given from previous incorrect result
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
sf	significant figure(s)
dp	decimal place(s)

Examiners should consistently apply the following general marking principles

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

Diagrams

Diagrams that have working on them should be treated like normal responses. If a diagram has been written on but the correct response is within the answer space, the work within the answer space should be marked. Working on diagrams that contradicts work within the answer space is not to be considered as choice but as working, and is not, therefore, penalised.

Work erased or crossed out

Erased or crossed out work that is still legible and has not been replaced should be marked. Erased or crossed out work that has been replaced can be ignored.

Choice

When a choice of answers and/or methods is given and the student has not clearly indicated which answer they want to be marked, mark positively, awarding marks for all of the student's best attempts. Withhold marks for final accuracy and conclusions if there are conflicting complete answers or when an incorrect solution (or part thereof) is referred to in the final answer.

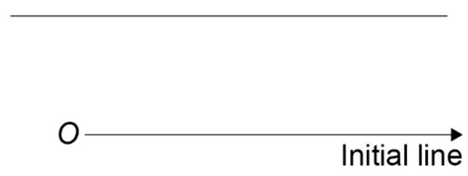
AS/A-level Maths/Further Maths assessment objectives

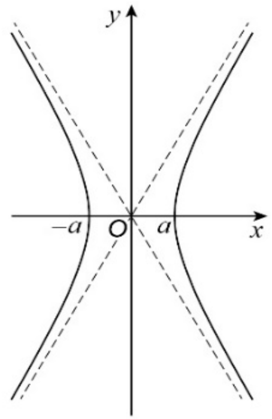
AO		Description
AO1	AO1.1a	Select routine procedures
	AO1.1b	Correctly carry out routine procedures
	AO1.2	Accurately recall facts, terminology and definitions
AO2	AO2.1	Construct rigorous mathematical arguments (including proofs)
	AO2.2a	Make deductions
	AO2.2b	Make inferences
	AO2.3	Assess the validity of mathematical arguments
	AO2.4	Explain their reasoning
	AO2.5	Use mathematical language and notation correctly
AO3	AO3.1a	Translate problems in mathematical contexts into mathematical processes
	AO3.1b	Translate problems in non-mathematical contexts into mathematical processes
	AO3.2a	Interpret solutions to problems in their original context
	AO3.2b	Where appropriate, evaluate the accuracy and limitations of solutions to problems
	AO3.3	Translate situations in context into mathematical models
	AO3.4	Use mathematical models
	AO3.5a	Evaluate the outcomes of modelling in context
	AO3.5b	Recognise the limitations of models
	AO3.5c	Where appropriate, explain how to refine models

Q	Marking instructions	AO	Marks	Typical solution
1	Circles correct answer	1.2	B1	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
	Total		**1**	

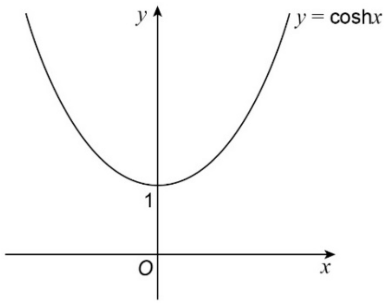
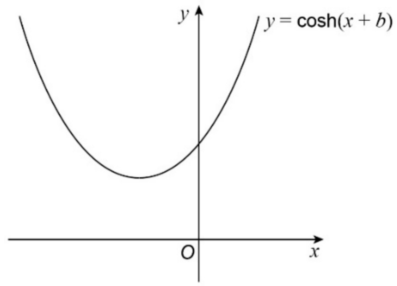
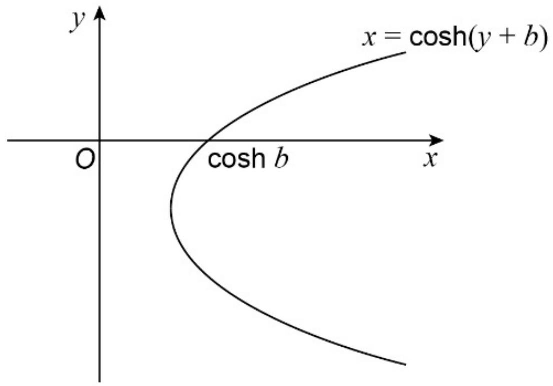
Q	Marking instructions	AO	Marks	Typical solution
2	Circles correct answer	1.1b	B1	$5a - 2b$
	Total		**1**	

Q	Marking instructions	AO	Marks	Typical solution
3	Circles correct answer	1.1b	B1	$\left(-1, \sqrt{3}\right)$
	Total		**1**	

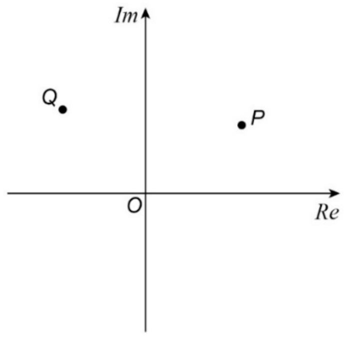
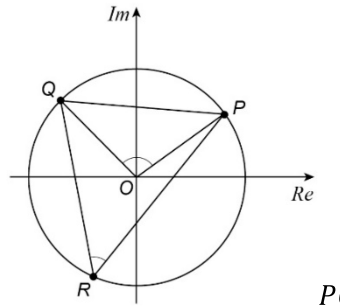
Q	Marking instructions	AO	Marks	Typical solution
4(a)	Draws a horizontal line above the point *O*, parallel to the initial line. Ignore a vertical axis through O. Ignore an extension of the initial line. Accept a freehand ‘straight’ line – mark intention. A deliberate curve, e.g. parabolic, is B0.	1.1b	B1	
4(b)	States the correct minimum distance as k Treat an answer of $k = 0$ as two responses, one correct and one incorrect (B0). Condone $r = k$ Ignore any value of θ, e.g. $\theta = \frac{\pi}{2}$ Must be simplified, e.g. $k - 0$ is B0, but $k - 0 = k$ is B1.	1.1b	B1	k
	Total		**2**	

Q	Marking instructions	AO	Marks	Typical solution
5(a)	Writes the correct equations with a removed. Accept any equivalent equations, e.g. $x = \pm\frac{1}{2}y$	1.1b	B1	$\frac{x}{a} = \pm\frac{y}{2a}$ $y = \pm 2x$
5(b)	Draws the correct graph, correctly approaching the asymptotes – mark the intention.	1.1b	B1	
	Writes $(a, 0)$ and $(-a, 0)$ Accept a and –a written close to the intercepts.	1.1b	B1	

Q	Marking instructions	AO	Marks	Typical solution
5(c)	Formulates an expression for a volume generated by rotating the hyperbola about an axis – must be a clear intent to integrate. A volume expression must be of the form $\int \mathrm{f}(x)$ or $\int \mathrm{f}(y)$ where f is a polynomial function of degree 2. Condone missing limits and/or π and/or $\mathrm{d}y$ (or dx).	3.1a	M1	$\text{Volume} = \pi \int x^2 \, \mathrm{d}y$ $= \pi \int_0^a \left(\frac{y^2}{4} + a^2\right) \mathrm{d}y$
	Expresses the volume as $\pi \int \left(\frac{y^2}{4} + a^2\right) \mathrm{d}y$ or equivalent. Must include π – may be seen later. Condone missing limits and/or dy.	1.1b	A1	
	Correctly integrates their expression. Their expression must be of the form $cy^2 + d$ or $cx^2 + d$ where c and d are constants.	1.1a	M1	$= \pi \left[\frac{y^3}{12} + a^2 y\right]_0^a$
	Substitutes correct limits into $py^3 + qy$, where p and q are positive constants. May be unsimplified. Accept substitution of 0 not seen.	1.1b	A1	$= \pi \left(\frac{a^3}{12} + a^3 - 0\right)$
	Completes a rigorous mathematical argument, including either ka^3 where $k \in [3.4005, 3.4045]$ or $\frac{13}{12}\pi a^3$ (or equivalent), that the volume can be expressed as $3.40a^3$ to 3 significant figures. Must use dy correctly throughout. Must include an appropriate reference to 3 significant figures, e.g. $\frac{13\pi}{12} = 3.40$ (3sf) Accept substitution of 0 not seen. This mark can only be awarded if M2A2 scored. NMS scores 0/5	2.1	R1	$= \frac{13\pi}{12} a^3$ $= 3.403a^3$ $= 3.40a^3$ (3 sig figs)
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
6(a)	Draws a ∪ shape or ⊂ or ∩ or ⊃ in any position. Condone a graph which appears to have an asymptote and/or a cusp.	1.2	B1	  
	Draws a ∪ shape in any orientation or position with the vertex **not** positioned on an axis. Condone a half graph drawn. Condone a graph which appears to have an asymptote and/or a cusp.	3.1a	M1	
	Draws a ⊂ shape. The line of symmetry must be parallel to the x-axis. Condone a half graph drawn, i.e. ╭ or ╰ Condone a graph which appears to have an asymptote and/or a cusp.	3.1a	M1	
	Draws a correct sketch of $x = \cosh(y+b)$ with x-intercept identified as $(\cosh b\,, 0)$ Ignore incorrect vertex coordinates. Do not condone a half graph, or a graph with asymptotes or a cusp. NMS can score 4/4	1.1b	A1	
6(b)	Deduces the correct minimum distance between the graph of $x = \cosh(y+b)$ and the y-axis as 1	2.2a	B1	1
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
7(a)	Obtains the required result with $A = 2$. Must show at least one intermediate step with no incorrect steps. Condone the LHS not appearing in their working.	1.1b	B1	$\frac{1}{r-1} - \frac{1}{r+1} \equiv \frac{r+1}{(r-1)(r+1)} - \frac{r-1}{(r-1)(r+1)} \equiv \frac{2}{r^2-1}$
7(b)	Writes at least five corresponding terms of $\frac{k}{r-1}$ and $\frac{k}{r+1}$ Must include the terms for $r = 2$, $r = 3$, $r = n-1$, $r = n$ and for either $r = 4$ or $r = n-2$ Condone $\frac{1}{0} - \frac{1}{2}$ also included.	1.1a	M1	$\sum_{r=2}^{n}\left(\frac{2}{r^2-1}\right) = \sum_{r=2}^{n}\left(\frac{1}{r-1} - \frac{1}{r+1}\right)$ $= \left(\frac{1}{1} - \frac{1}{3}\right) + \left(\frac{1}{2} - \frac{1}{4}\right) + \left(\frac{1}{3} - \frac{1}{5}\right) + \ldots\ldots\ldots$ $+ \left(\frac{1}{n-3} - \frac{1}{n-1}\right) + \left(\frac{1}{n-2} - \frac{1}{n}\right) + \left(\frac{1}{n-1} - \frac{1}{n+1}\right)$
	Correctly uses the method of differences to reduce the expression to four terms, or equivalent.	1.1b	A1	$= \frac{1}{1} + \frac{1}{2} - \frac{1}{n} - \frac{1}{n+1}$
	Multiplies by $\frac{1}{2}$ (may be seen at any stage).	1.1a	M1	$\sum_{r=2}^{n}\left(\frac{1}{r^2-1}\right) = \frac{1}{2}\left(\frac{3}{2} - \frac{(n+1)+n}{n(n+1)}\right)$
	Completes fully correct working to reach the required result. This mark is only available if all previous marks have been awarded.	2.1	R1	$= \frac{3}{4} - \frac{2n+1}{2n(n+1)}$ $= \frac{3(n^2+n) - 2(2n+1)}{4n(n+1)}$ $= \frac{3n^2 - n - 2}{4n(n+1)}$
	Total		**5**	

Q	Marking instructions	AO	Marks	Typical solution
8(a)	States 4	1.1b	B1	4
8(b)	Obtains $-\frac{7\pi}{12}$ Accept $-1.83(2595715\ldots)$	1.1b	B1	$\frac{\pi}{6}-\frac{3\pi}{4}=-\frac{7\pi}{12}$
8(c)	Draws a point (or line from O) labelled P (or z_1) in the first quadrant.	1.1b	B1	
	Draws a point (or line from O) labelled Q (or z_2) in the second quadrant.	1.1b	B1	
8(d)	Recognises that *P*, *Q* and *R* are points on the circumference of a circle – possibly implied by circle drawn on diagram. Or recognises $\|w\| = 2$ is a circle.	3.1a	B1	 $P\hat{O}Q=\frac{3\pi}{4}-\frac{\pi}{6}=\frac{7\pi}{12}$ $P\hat{R}Q=\frac{7\pi}{12}\div 2$ (angle at circumference $=\frac{1}{2}\times$ angle at centre) $=\frac{7\pi}{24}$
	Correctly deduces the angle $P\hat{R}Q$ as $\frac{7\pi}{24}$ Accept any exact equivalent angle, e.g. $0.291\dot{6}\pi$ or 52.5°	2.2a	B1	
	Explains why the angle $P\hat{R}Q$ is half that of the angle $P\hat{O}Q$ for any position of R. Award 3/3 for a complete and correct algebraic solution.	2.4	E1	
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
9(a)	Identifies the 2nd line or $1-\cosh^2 x$ as the error. Possibly implied by the explanation.	2.3	B1	In the 2nd line, $\sinh^2 x$ should not have been replaced with $1-\cosh^2 x$
	Explains that $\sinh^2 x$ should not be replaced with $1-\cosh^2 x$ Accept any correct arrangement of the identity $\cosh^2 x - \sinh^2 x = 1$	2.4	E1	
9(b)	Uses a correct identity to express the equation in terms of one function. e.g. uses $\cosh^2(2x) - \sinh^2(2x) = 1$ to write the equation in terms of $\cosh(2x)$ only	1.1b	B1	$\cosh^2(2x) - 1 - 2\cosh(2x) = 1$
	Rearranges their equation into a quadratic=0 or quartic=0 Must be in terms of just one function. Possibly implied by a correct value for their function.	1.1a	M1	$\cosh^2(2x) - 2\cosh(2x) - 2 = 0$
	Finds a correct exact value for their function, e.g. $\cosh x = \pm\sqrt{1+\frac{\sqrt{3}}{2}}$ ISW incorrect work following a correct unsimplified answer.	1.1b	A1	$\cosh(2x) = 1 \pm \sqrt{3}$
	Correctly explains why they reject at least one of their solutions, e.g. $\cosh 2x < 1$ cannot be a solution. Follow through B1M1 only.	2.4	E1F	but $\cosh(2x) \geq 1 \therefore \cosh(2x) = 1+\sqrt{3}$
	Finds both correct sets of values for p, q and r and no others. Accept $\pm\frac{1}{2}\cosh^{-1}(1+\sqrt{3})$ for full marks.	1.1b	A1	$x = \pm\frac{1}{2}\cosh^{-1}(1+\sqrt{3})$ $p = \pm 2$, $q = 1$, $r = 3$
	Total		**7**	

Q	Marking instructions	AO	Marks	Typical solution
10(a)	Uses, or writes, $\cosh x = \frac{1}{2}(e^x + e^{-x})$	1.2	B1	$\cosh x = \frac{1}{2}(e^x + e^{-x})$
	Substitutes into their $\cosh x$ the Maclaurin expansions of e^x and e^{-x} up to x^3 or beyond. Allow one sign error. Ignore errors beyond x^4	1.1a	M1	$\cosh x = \frac{1}{2}\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\right) + \frac{1}{2}\left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots\ldots\right)$
	Correctly simplifies their expression to $1 + \frac{x^2}{2} + \frac{x^4}{24}$ or equivalent. Accept 2! for 2 and 4! for 24. Ignore terms beyond x^4 NMS can score 3/3	1.1b	A1	$\cosh x = 1 + \frac{x^2}{2} + \frac{x^4}{24}$
10(b)	Substitutes ix for x in their expansion of $\cosh x$	1.1a	M1	$\cosh(ix) = 1 + \frac{(ix)^2}{2} + \frac{(ix)^4}{24}$
	Correctly simplifies the powers of i to give $1 - \frac{x^2}{2} + \frac{x^4}{24}$ or equivalent. Accept 2! for 2 and 4! for 24. Ignore terms beyond x^4 Implied by a correct answer of $\cos x$	1.1b	A1	$= 1 + \frac{-1x^2}{2} + \frac{1x^4}{24}$
	Recognises the Maclaurin expansion of $\cos x$ NMS can score 3/3	1.2	B1	$= \cos x$
	Total		**6**	

Q	Marking instructions	AO	Marks	Typical solution
11(a)	Gives $x = \sqrt{r}$ or $x = -\sqrt{r}$ or $y = 1$ as an asymptote. Condone other incorrect asymptotes.	1.1b	B1	$x^2 - r = 0$ $x^2 = r$ $x = \pm\sqrt{r}$
	Gives $x = \pm\sqrt{r}$ and $y = 1$ as asymptotes, with no incorrect asymptotes given.	1.1b	B1	$y = 1$
11(b)	Rearranges $k = \frac{x^2+x-6}{x^2-1}$ into a non-fractional form. Accept any sensible alternative for k, e.g. y or f	3.1a	M1	Let $k = \frac{x^2+x-6}{x^2-1}$ $k(x^2-1) = x^2 + x - 6$
	Rearranges their equation into a correct three-term quadratic equation in x Condone missing $= 0$ Possibly implied by a correct discriminant.	1.1b	A1	$(k-1)x^2 - x + 6 - k = 0$
	Correctly substitutes their coefficients into $b^2 - 4ac$ to obtain an expression in k only. Accept any sensible alternative for k, e.g. y or f	3.1a	M1	
	Obtains a correct quadratic equation/inequality in k – may be unsimplified. Or obtains the correct critical values of k. Accept any sensible alternative for k, e.g. y or f	1.1b	A1	$1 - 4(k-1)(6-k) \geq 0$
	Obtains the correct critical values. Accept non-exact values to at least 3 sig figs, e.g. 1.05 and 5.95	1.1b	A1	
	Gives a correct range in terms of y using exact values. Condone 'and'. Follow through their critical values if M2 scored and quadratic inequality seen. Do not accept an alternative for y Accept any equivalent expressions for $\frac{7-2\sqrt{6}}{2}$ and $\frac{7+2\sqrt{6}}{2}$ NMS scores 0/6	2.2a	A1F	$1 - 4(6k - k^2 - 6 + k) \geq 0$ $4k^2 - 28k + 25 \geq 0$ $y \leq \frac{7-2\sqrt{6}}{2}$, $y \geq \frac{7+2\sqrt{6}}{2}$
	Total		**8**	

Q	Marking instructions	AO	Marks	Typical solution
12(a)	Demonstrates the rule is correct for $n = 1$ and states that it is true for $n = 1$ (may appear at any stage).	1.1b	B1	Try $n = 1$: $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}^1 = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ and $\begin{bmatrix}1 & 3^1 - 1\\0 & 3^1\end{bmatrix} = \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ $\therefore$ true for $n = 1$
	Multiplies $\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ and $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ Accept any letter in place of k (condone n).	2.4	M1	Assume true for $n = k$ $\therefore\ \mathbf{A}^k = \begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ $\Rightarrow\ \mathbf{A}^k \times \mathbf{A} = \begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix} \times \begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$
	Obtains $\begin{bmatrix}1 & 3^{k+1} - 1\\0 & 3^{k+1}\end{bmatrix}$ from multiplying $\begin{bmatrix}1 & 3^k - 1\\0 & 3^k\end{bmatrix}$ and $\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}$ Must include an intermediate step for the top right element.	2.2a	A1	$\Rightarrow\ \mathbf{A}^{k+1} = \begin{bmatrix}1 & 2 + 3(3^k - 1)\\0 & 3^k \times 3\end{bmatrix}$ $= \begin{bmatrix}1 & 3^{k+1} - 1\\0 & 3^{k+1}\end{bmatrix}$
	Completes a rigorous argument and explains how their argument proves the required result. e.g. states “assume that the rule is true for $n = k$” (or equivalent) and “also true for $n = k + 1$” (or equivalent) and “for all n” and includes the base case with a conclusion. Do not accept the use of n in place of k. NMS scores 0/4	2.1	R1	$\therefore$ it is also true for $n = k + 1$ True for $n = 1$, and true for $n = k\ \Rightarrow$ true for $n = k + 1$ Then, by induction, it is true for all integers $n \geq 1$

Q	Marking instructions	AO	Marks	Typical solution
12(b)	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Correctly substitutes $y = mx + c$ and $y' = mx' + c$	1.1b	A1	$x' = x + 2(mx + c)$ and $mx' + c = 3(mx + c)$
	Eliminates one variable to leave an equation in m, c and just one other variable.	1.1a	M1	$m(x + 2mx + 2c) + c \equiv 3mx + 3c$
	Compares coefficients to produce two correct equations in m and c	1.1b	A1	$m + 2m^2 = 3m$ and $2mc + c = 3c$
	Gives $y = 0$ or $y = x + c$ as invariant lines. Condone other incorrect invariant lines.	1.1b	B1	$m(m-1) = 0$ and $c(m-1) = 0$ $m = 0$ or $m = 1$ and $c = 0$ or $m = 1$
	Gives $y = 0$ and $y = x + c$ as invariant lines, with no incorrect invariant lines. NMS can score 2/6	2.2a	B1	$y = 0x + 0$ or $y = 1x + c$ Invariant lines are $y = 0$ and $y = x + c$
Q12b: Alternative mark scheme for students who assume that all invariant lines pass through the origin – max 3 marks				
Q12b ALT	Sets up two equations in (x, y) and its image (x', y')	3.1a	M1	$\begin{bmatrix}1 & 2\\0 & 3\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix} = \begin{bmatrix}x'\\y'\end{bmatrix}$ $x' = x + 2y$ and $y' = 3y$
	Eliminates three variables to leave an equation in m and just one other variable.	1.1a	M1	$x' = x + 2(mx)$ and $mx' = 3(mx)$ $m(x + 2mx) \equiv 3mx$
	Gives $y = 0$ and $y = x$ as invariant lines, with no other incorrect invariant lines. NMS can score 1/3	2.2a	B1	$m + 2m^2 = 3m$ $m(m-1) = 0$ $m = 0$ or $m = 1$ $y = 0x$ or $y = 1x$ Invariant lines are $y = 0$ and $y = x$

Q	Marking instructions	AO	Marks	Typical solution
12(c)	Sets up at least one correct equation in x and y Accept alternative variables for this mark.	1.1a	M1	$x = x + 2y$ and $y = 3y$
	Gives $y = 0$ as the only line of invariant points. NMS can score 2/2	1.1b	A1	$y = 0$
	Total		**12**	

Q	Marking instructions	AO	Marks	Typical solution
13(a)	Rewrites l_1 in the general Cartesian form, or as a vector in terms of just one parameter. Or finds the position vector of a point on the line. Or finds a direction vector of l_1 Or writes three equations expressing x, y and z in terms of the parameter.	3.1a	M1	$\frac{x-3}{1}=\frac{y+1}{1.5}=\frac{z-2}{-1}$
	Writes l_1 in a correct vector form. Accept $\begin{pmatrix}x\\y\\z\end{pmatrix}$ in place of $\mathbf{r}$ NMS can score 2/2	1.1b	A1	$\mathbf{r}=\begin{pmatrix}3\\-1\\2\end{pmatrix}+\lambda\begin{pmatrix}1\\1.5\\-1\end{pmatrix}$
13(b)(i)	Calculates the scalar product of their direction vectors of lines l_1 and l_2 Must not use a position vector as a direction vector.	3.1a	M1	$\begin{pmatrix}1\\1.5\\-1\end{pmatrix}.\begin{pmatrix}n-3\\3\\n\end{pmatrix}=n-3+4.5-n=1.5$
	Calculates a correct scalar product of a vector parallel to $\begin{pmatrix}1\\1.5\\-1\end{pmatrix}$ with a vector parallel to $\begin{pmatrix}n-3\\3\\n\end{pmatrix}$	1.1b	A1	
	Explains that, as the scalar product is not zero, then the lines are not perpendicular. Follow through their direction vectors if their scalar product is non-zero. Must follow M1. NMS scores 0/3	3.2a	E1F	The scalar product is not zero $\therefore$ lines l_1 and l_2 are not perpendicular

Q	Marking instructions	AO	Marks	Typical solution
13(b)(ii)	Explains that two vectors are parallel if one is a multiple of the other. l_1 and l_2 need not be referred to explicitly. Possibly implied by $\boldsymbol{c} = k\boldsymbol{d}$ seen where $\boldsymbol{c}$ and $\boldsymbol{d}$ are their direction vectors.	2.4	E1F	If $\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} n-3 \\ 3 \\ n \end{pmatrix}$ are parallel then $n - 3 = 2$ and $n = -2$
	Demonstrates that, as n varies, the two direction vectors can never be a multiple of each other.	3.1a	B1	but n cannot be both 5 and -2 $\therefore$ l_1 and l_2 cannot be parallel
13(b)(iii)	Uses the scalar product to form an equation in n and $\cos\theta$ Follow through their direction vectors for lines l_1 and l_2	1.1a	M1	$\begin{pmatrix} 2 \\ 3 \\ -2 \end{pmatrix} . \begin{pmatrix} n-3 \\ 3 \\ n \end{pmatrix} = \sqrt{2^2 + 3^2 + 2^2} \times \sqrt{(n-3)^2 + 3^2 + n^2} \times \cos\theta$
	Forms a correct equation in n and $\cos\theta$ Or gives a correct expression for $\cos\theta$ Accept a correct equation, or expression, for the supplementary angle.	1.1b	A1	$2(n-3) + 9 - 2n = \sqrt{17} \times \sqrt{2n^2 - 6n + 18} \times \cos\theta$
	Writes $\cos\theta = \frac{3}{\sqrt{34n^2 - 102n + 306}}$	2.1	R1	$\cos\theta = \dfrac{3}{\sqrt{34n^2 - 102n + 306}}$
	Total		**10**	

Q	Marking instructions	AO	Marks	Typical solution
14(a)	Gives a full and correct explanation. Accept $\alpha+\beta+\gamma=0$ followed by $\alpha+\beta=-\gamma$ without further justification.	2.4	E1	The coefficient of x^2 is zero $\therefore\ \alpha+\beta+\gamma=0$ $\alpha+\beta=-\gamma$
14(b)	Substitutes –1 or 1 into x^3-3x Possibly implied by -2 or 2	3.1a	M1	max point $=(-1,2)$ and min point $=(1,-2)$
	Finds the correct set of values for p. Condone 'and'. NMS scores 2/2	2.2a	A1	$p<-2\ ,\ p>2$
14(c)(i)	Correctly expands $(\alpha+1)(\beta+1)(\gamma+1)$ Or recognises that $y=\mathrm{f}(x)$ is a horizontal translation of $y=x^3-3x+p$ (possibly implied by the sight of $x+1$ or $x-1$) Accept any sensible alternative for x.	3.1a	B1	Let $w=x+1$ $x=w-1$
	Substitutes 0 for $\alpha+\beta+\gamma$, and ±3 for $\alpha\beta+\beta\gamma+\gamma\alpha$, and $\pm p$ for $\alpha\beta\gamma$ Or substitutes $x-1$ for x in x^3-3x+p Accept any sensible alternative for x.	1.1a	M1	$(w-1)^3-3(w-1)+p=0$
	Shows the required result. NMS scores 0/3	2.1	R1	$w^3-3w^2+3w-1-3w+3+p=0$ $w^3-3w^2+p+2=0$ constant term $=p+2$
14(c)(ii)	Gives the correct x-intercepts and no others.	2.2a	B1	0 and 2
	Total		**7**	
	PAPER TOTAL		**80**	