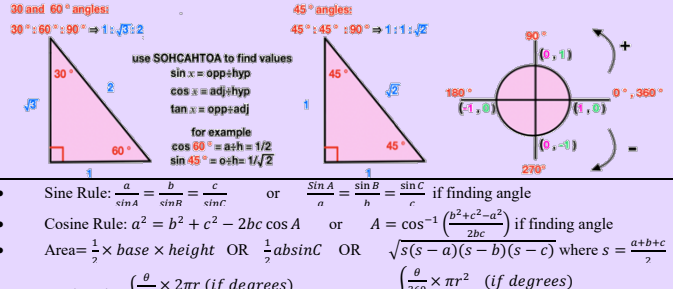
Name	Formula	Comments, Common Mistakes And When To Use
Quotient	• $\tan x = \frac{\sin x}{\cos x}$ • $\cot x = \frac{\cos x}{\sin x}$	The formulae hold for any angle as long as they are the same: $tan2x = \frac{\sin 2x}{\cos 2x}$ etc
Reciprocal	• $\sec x = \frac{1}{\cos x} \Leftrightarrow \cos x = \frac{1}{\sec x}$ • $\operatorname{cosec} x = \frac{1}{\sin x} \Leftrightarrow \sin x = \frac{1}{\operatorname{cosec} x}$ • $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x} \Leftrightarrow \tan x = \frac{1}{\cot x}$	The formulae hold for any angles as long as they are the same: $sec2x = \frac{1}{cos2x}$ etc
Quadratic (Pythagorean)	• $sin^2 x + cos^2 x = 1$ ↳ $\mathbf{cos^2 x = 1 - sin^2 x}$ ↳ $\mathbf{sin^2 x = 1 - cos^2 x}$ • $1 + tan^2 x = sec^2 x.$ ↳ $\mathbf{tan^2 x = sec^2 x - 1}$ • $1 + cot^2 x = cosec^2 x$ ↳ $\mathbf{cot^2 x = cosec^2 x - 1}$	• Re-arrangements are just as important as the original formula!! • Proof: Divide all of $sin^2 x + cos^2 x = 1$ by $\sin^2 x$ or $\cos^2 x$ to get 2nd and 3rd formulae • When solving and proving identities, don't forget you can replace $sin^2 x + cos^2 x$ with 1 and vice versa • The formulae hold for any angles as long as they're the same: $sin^2 2x + cos^2 2x = 1$ etc
Double Angle	• $\sin 2x = 2 \sin x \cos x$ • $cos2x = cos^2 x - sin^2 x$ $= 2\cos^2 x - 1$ ↳ $\mathbf{cos^2 x = \frac{1}{2}(1 + cos2x)}$ $= 1 - 2\sin^2 x$ ↳ $\mathbf{sin^2 x = \frac{1}{2}(1 - cos2x)}$ • $\tan 2x = \frac{2\tan x}{1-tan^2 x}$ ↳ $\mathbf{tan^2 x = \frac{1-\cos 2x}{1+\cos 2x}}$	• Re-arrangements are just as important as the original formula!! • Proof: write angles as $a + a$ and use addition formula etc • The formulae hold for any doubling relationship between the angles. Notice how the coefficients don't change $\sin 4x = 2\sin 2x \cos 2x$ $\cos 4x = 1 - 2\sin^2 2$ *or* $2\cos^2 2x - 1$ or $\cos^2 2x - \sin^2 2x$ etc
Half Angle	• $sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$ • $cos\frac{x}{2} = \pm\sqrt{\frac{\cos x+1}{2}}$ • $tan\frac{x}{2} = \pm\sqrt{\frac{1-cosx}{1+cos x}} = \frac{1-cosx}{sinx} = \frac{sin x}{1+cos x}$ Formulae with $t = \tan\left(\frac{x}{2}\right)$: • $\sin x = \frac{2t}{1+t^2}$ • $\cos x = \frac{1-t^2}{1+t^2}$ • $\tan x = \frac{2t}{1-t^2}$	• Proof: Use double angle $cos2x = 2\cos^2 x - 1$ $cos2x = 1 - 2\sin^2 x$ $cosx = 2\cos^2\frac{x}{2} - 1$ $cosx = 1 - 2\sin^2\frac{x}{2}$ $cos\frac{x}{2} = \pm\sqrt{\frac{\cos x+1}{2}}$ $sin\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{2}}$
Triple Angle & Powers of Trig	$sin 3x = 3\sin x - 4\sin^3 x$ ↳ $\mathbf{\sin^3 x = \frac{1}{4}(3\sin x - \sin 3x)}$ $cos 3x = 4\cos^3 x - 3\cos x$ ↳ $\mathbf{cos^3 x = \frac{1}{4}(3\cos x + \cos 3x)}$ $tan 3x = \frac{3\tan x - \tan^3 x}{1-3\tan^2 x}$	• Proof: write angles as $2a + a$ and use addition formula etc. • Other powers of trig by finding $sin\,4x$, $\sin 5x$ etc: $sin^4 x = \frac{1}{8}(\cos 4x - 4\cos 2x + \frac{6}{2})$ and $cos^4 x = \frac{1}{8}(\cos 4x + 4\cos 2x + \frac{6}{2})$ $sin^5 x = \frac{1}{16}(\sin 5x - 5\sin 3x + 10\sin x)$ and $cos^5 x = \frac{1}{4}(\cos 5x + 5\cos 3x + 10\cos x)$
Cofunction	• $\cos x = \sin(90 - x)$ and $\sin x = \cos(90 - x)$ • $\tan x = \cot(90 - x)$ and $\cot x = \tan(90 - x)$ • $\sec x = \operatorname{cosec}(90 - x)$ and $\operatorname{cosec} x = \sec(90 - x)$	These are useful when you want to swap sin to cos and vice versa. Using $\sin x = \pm\sqrt{1 - \cos^2 x}$ *and* $\cos x = \pm\sqrt{1 - \sin^2 x}$ from Pythag identities make things more complicated, so it's better to replace $\sin x$ with $\cos(90 - x)$ etc
Even Odd	• $\sin(-x) = -sinx$ and $\cos(-x) = \cos x$ • $\tan(-x) = -\tan x$ and $\cot(-x) = -\cot x$ • $\sec(-x) = \sec x$ and $\operatorname{cosec}(-x) = -cosec\,x$	**© MyMathsCloud**
Addition	• $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$ • $cos(A \pm B) = cosAcosB \mp sinAsinB$ • $tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$	• To turn $a\sin x \pm b\cos x$ into form $r\sin(x \pm \alpha)$ or $a\cos x \pm b\cos x$ into form $r\cos(x \mp \alpha)$: r: $\sqrt{a^2 + b^2}$ α: $tan^{-1}\left(\left\|\frac{b}{a}\right\|\right)$ and then draw this angle in the quadrant where the coordinate (a,b) lies Read off α by starting on the x axis (remember: $-\pi \le \alpha < \pi$)
Product to sum (Factor Formula)	• $\sin x \sin y = \frac{1}{2}[\cos(x - y) - \cos(x + y)]$ and $\cos x \cos y = \frac{1}{2}[\cos(x - y) + \cos(x + y)]$ $\sin x \cos y = \frac{1}{2}[\sin(x + y) + \sin(x - y)]$ and $\cos x \sin y = \frac{1}{2}[\sin(x + y) - \sin(x - y)]$	• Used mainly when you want to integrate products • Proof: Add or subtract 2 pairs or addition angle formulae and solve simultaneously *eg.* $\sin(A + B) = \sin A \cos B + \cos A \sin B$ and $\sin(A - B) = \sin A \cos B - \cos A \sin B$ add to get $\sin(A + B) + \sin(A - B) = 2\sin A \cos B$ dividing by 2 gives $\sin A \cos B = \frac{1}{2}[\sin(A + B) + \sin(A - B)]$
Sum to product (Factor Formulae)	• $sinA + sinB \equiv 2sin\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)$ and $sinA - sinB \equiv 2cos\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right)$ • $cosA + cosB \equiv 2cos\left(\frac{A+B}{2}\right)cos\left(\frac{A-B}{2}\right)$ and $cosA - cosB \equiv -2sin\left(\frac{A+B}{2}\right)sin\left(\frac{A-B}{2}\right)$	• Proof: Let $\alpha = \frac{A+B}{2}, \beta = \frac{A-B}{2} \Rightarrow \alpha + \beta = \frac{A+B}{2} + \frac{A-B}{2} = A$ and $\alpha - \beta = \frac{A+B}{2} + \frac{A-B}{2} = B$ e.g. $\sin\alpha\cos\beta = \frac{1}{2}[\sin(\alpha + \beta) + \sin(\alpha - \beta)]$ Replace α and β in product to sum formula $\Rightarrow \sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) = \frac{1}{2}[\sin A + \sin B]$
Small Angle	$\sin\theta \approx \theta$ $\cos\theta \approx 1 - \frac{\theta^2}{2}$ $\tan\theta \approx \theta$	Useful when finding limits as $x \to 0$
Inverse	Notation: $\sin^{-1} x = \arcsin x$ and $\sin^{-1} x$ is equivalent to $\sin\theta = x$ (etc for other trig function) • $sin^{-1}x + \cos^{-1} x = \frac{\pi}{2}$ and $\tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ and $\operatorname{cosec}^{-1} x + \sec^{-1} x = \frac{\pi}{2}$ • $\sin(sin^{-1}x) = x$ and $\sin^{-1}(sinx) = x$ (etc for other trig functions) • $tan^{-1} x \pm \tan^{-1} y = \tan^{-1}\left(\frac{x \pm y}{1 - xy}\right)$	
Differentiation	$sinf(x) \Rightarrow \boldsymbol{f'(x)}cosf(x)$ $cosf(x) \Rightarrow -\boldsymbol{f'(x)}sinf(x)$ $tanf(x) \Rightarrow \boldsymbol{f'(x)}sec^2 f(x)$ $secf(x) \Rightarrow \boldsymbol{f'(x)}secf(x)tanf(x)$ $cosecf(x) - \boldsymbol{f'(x)}cosecf(x)cotf(x)$ $cotf(x) \Rightarrow -\boldsymbol{f'(x)}cosec^2 f(x)$ $sin^{-1} f(x) \Rightarrow \frac{f'(x)}{\sqrt{1-(f(x))^2}}$ $cos^{-1} f(x) \Rightarrow -\frac{f'(x)}{\sqrt{1-(f(x))^2}}$ $tan^{-1} f(x) \Rightarrow \frac{f'(x)}{1+(f(x))^2}$ $sec^{-1} f(x) \Rightarrow \frac{f'(x)}{f(x)\sqrt{(f(x))^2-1}}$ $cosec\,f(x) = \Rightarrow -\frac{f'(x)}{f(x)\sqrt{(f(x))^2-1}}$ $cot^{-1} f(x) \Rightarrow -\frac{f'(x))}{1+(f(x))^2}$	**Graphing** $y = \sin x$: Domain: $x \in \mathbb{R}$; Range: $-1 \le y \le 1$; Period: 2π $y = \cos x$: Domain: $x \in \mathbb{R}$; Range: $-1 \le y \le 1$; Period: 2π $y = \tan x$: Domain: $x \in \mathbb{R}, x \ne \frac{\pi}{2} + n\pi$; Range: $y \in \mathbb{R}$; Period: π $y = \operatorname{cosec} x$ $y = \sec x$ $y = \cot x$ $y = \sin^{-1} x$ $y = \cos^{-1} x$ $y = \tan^{-1} x$
Integration	$\int f'(x)\sin f(x)dx \Rightarrow -\cos f(x)$ $\int f'(x)\cos f(x)dx \Rightarrow \sin f(x)$ Integrate / Differentiate: S, -C, C, -S $\int f'(x)\sec^2 f(x)\,dx \Rightarrow \tan f(x)$ $\int f'(x)\sec f(x)\tan f(x)\,dx \Rightarrow \sec f(x)$ $\int f'(x)\operatorname{cosec} f(x)\cot f(x)\,dx \Rightarrow -\operatorname{cosec} f(x)$ $\int f'(x)\operatorname{cosec}^2 f(x)\,dx = -\cot f(x)$ $\int cosec\,f(x)\,dx = -\ln\lvert\operatorname{cosec} f(x) + \cot f(x)\rvert$ $\int \sec f(x)\,dx = \ln\lvert\sec f(x) + \tan f(x)\rvert$ $\int tan\,f(x)\,dx = \ln\lvert\sec f(x)\rvert$ $\int cot\,f(x)\,dx = \ln\lvert\sin f(x)\rvert$ $\int \frac{1}{\sqrt{a^2-(bx)^2}}dx \Rightarrow \frac{1}{b}\sin^{-1}\left(\frac{bx}{a}\right)$ $\int \frac{1}{-\sqrt{a^2-(bx)^2}}dx \Rightarrow \frac{1}{b}\cos^{-1}\left(\frac{bx}{a}\right)$ $\int \frac{1}{a^2+(bx)^2}\,dx \Rightarrow \frac{1}{ab}\tan^{-1}\left(\frac{bx}{a}\right)$	**To find equation:** $\boldsymbol{y = asin(bx + c)+d}$ **or** $\boldsymbol{y = acos(bx + c)+d}$ a = amplitude$=\frac{\max y - \min y}{2}$, $b = \frac{2\pi}{period}$ or $\frac{360}{period}$, d = principal axis $=\frac{\max y + \min y}{2}$, c =phase shift (plug in point to find)
Special Angles		
Triangle/Sector	• Sine Rule: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ if finding angle • Cosine Rule: $a^2 = b^2 + c^2 - 2bc\cos A$ or $A = \cos^{-1}\left(\frac{b^2+c^2-a^2}{2bc}\right)$ if finding angle • Area= $\frac{1}{2} \times base \times height$ OR $\frac{1}{2}absinC$ OR $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ • Arc length= $\begin{cases}\frac{\theta}{360} \times 2\pi r & (if\ degrees)\\ r\theta & (if\ radians)\end{cases}$ Arc Area= $\begin{cases}\frac{\theta}{360} \times \pi r^2 & (if\ degrees)\\ \frac{1}{2}r^2\theta & (if\ radians)\end{cases}$	We use sine, cosine rule and area formulae for non-right angled trig (they work for right angles too, but not necessary to use). For right angled trig we use SOHCAHTOA: "Some Old Hags Can't Always Hide Their Old Age" $\sin x = \frac{opp}{hyp}$, $\cos x = \frac{adj}{hyp}$, $\tan x = \frac{opp}{adj}$ and area= $\frac{1}{2} \times base \times height$ Use sine rule when we have a complete pairing of a side and an angle, meaning we have the numerical value of angle and its side opposite. If we don't, then we use cosine rule
Complex Numbers	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$ where $\lvert z\rvert = r$ and $\arg z = \theta$	De Moive's Theorem: $z^n = r^n(\cos n\theta + i\ \sin n\theta)$

	$\sin^{-1} x$	$\cos^{-1} x$	$\tan^{-1} x$
Domain	$-1 \le x \le 1$	$-1 \le x \le 1$	$-\infty \le x \le \infty$
Range	$-\frac{\pi}{2} \le x \le \frac{\pi}{2}$	$0 \le x \le \pi$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$

	0	30°	45°	60°	90°	180°	270°	360°
$\sin x$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos x$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
$\tan x$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0