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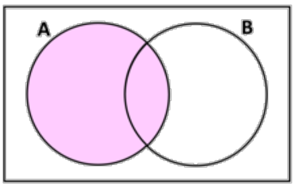
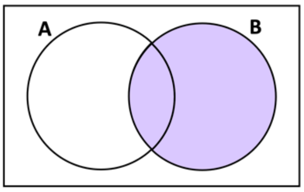
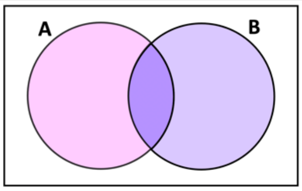
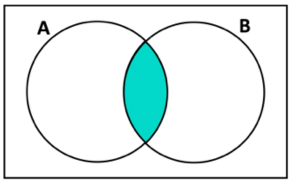
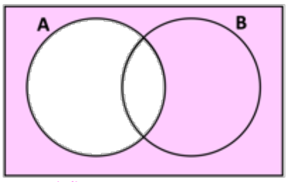
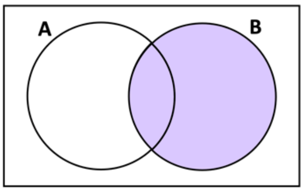
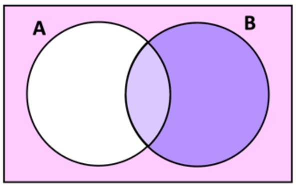
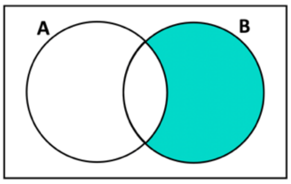
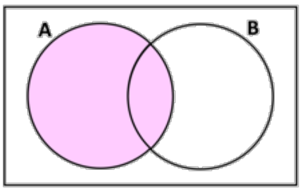
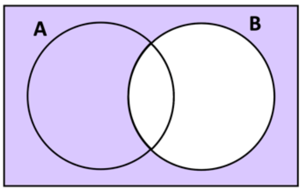
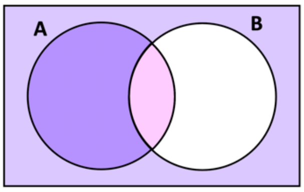
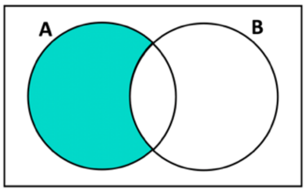
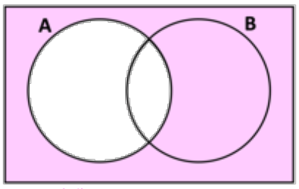
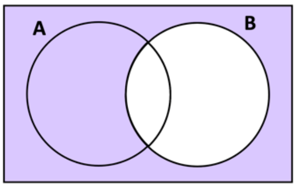
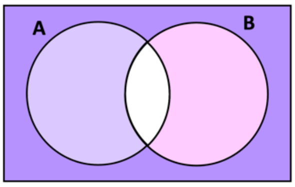
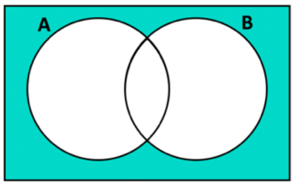
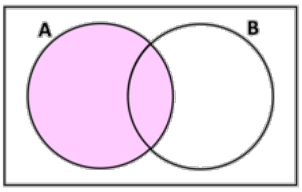
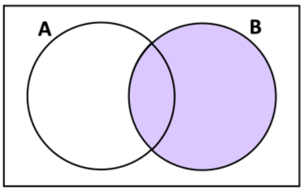
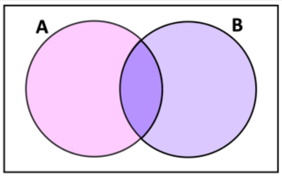
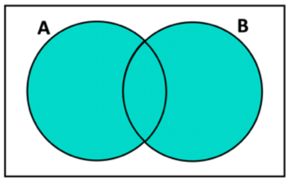
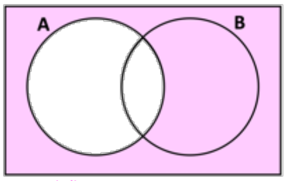
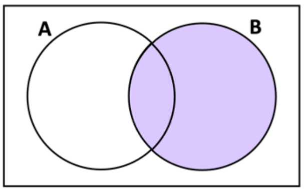
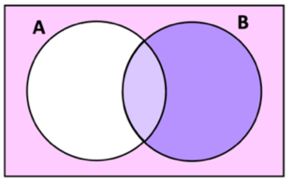
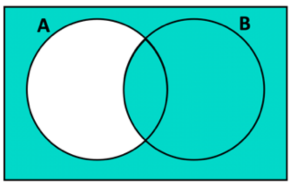
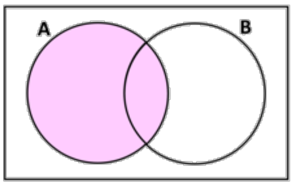
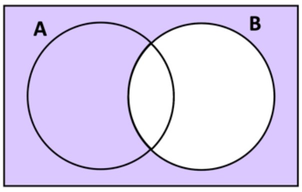
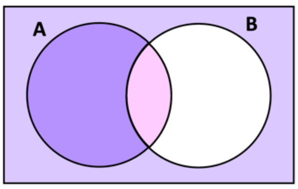
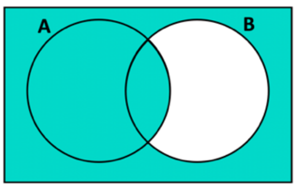
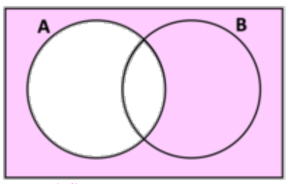
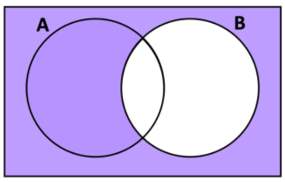
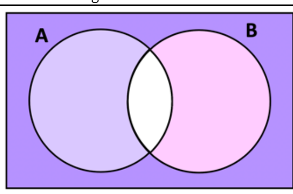
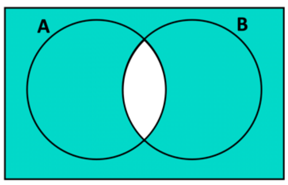
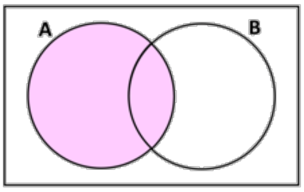
How to deal with an ∪ (**union** / **or**) versus an ∩ (**intersection**/ **and**)

	Step 1: (Shade the LHS)	Step 2: (Shade the RHS)	Step 3: Put Steps 1 and 2 Together	Step 4: Deal with the Symbol (This is the answer)
$\boldsymbol{P(A \cap B)}$	 P(A) = Everything in A	 P(B) = Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams (overlap)
$\boldsymbol{P(A' \cap B)}$ Note: $'$ means not	 $P(A')$ = Everything NOT in A	 $P(B)$ = Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams (overlap)
$\boldsymbol{P(A \cap B')}$ Note: $'$ means not	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams (overlap)
$\boldsymbol{P(A' \cap B')}$ Note: $'$ means not	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∩ (means and) so we take what was shaded in **BOTH** diagrams i.e. the double shading **common** to both diagrams (overlap)
$\boldsymbol{P(A \cup B)}$	 $P(A)$ =Everything in A	 $P(B)$ =Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
$\boldsymbol{P(A' \cup B)}$ Note: $'$ means not	 $P(A')$ = Everything NOT in A	 $P(B)$ =Everything in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
$\boldsymbol{P(A \cup B')}$ Note: $'$ means not	 P(A) = Everything in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded
$\boldsymbol{P(A' \cup B')}$ Note: $'$ means not	 $P(A')$ = Everything NOT in A	 $P(B')$ = Everything NOT in B	 The darker region indicates the overlap where the regions were shaded twice	 ∪ (means or) so we **merge/combine** everything that is shaded

Results

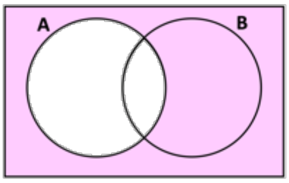
From the above you should now be comfortable all the following (you can either memorise them or use the method mentioned above to find them).

$P(A)$



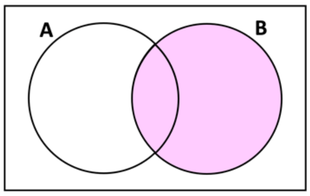
In English: This says all of A

$P(A')$



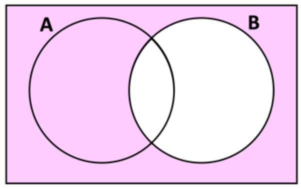
In English: This says everything that isn't in A

$P(B)$



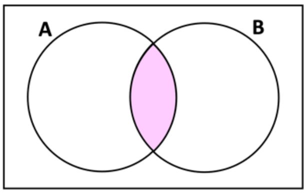
In English: This says all of B

$P(B')$



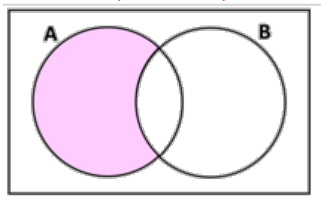
In English: This says everything that isn't in B

$P(A \cap B)$



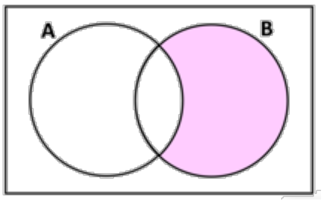
In English: This says everything that is in AND at the same time must be in B

$P(A \cap B')$



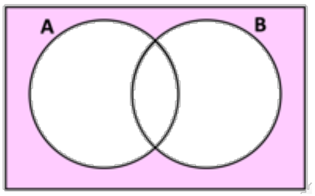
In English: This says everything that is A AND at the same time must not be in B.

$P(A' \cap B)$



In English: This says everything that is not in A AND at the same time must not be in B.

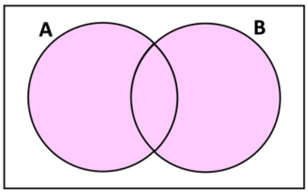
$P(A' \cap B')$



In English: This says everything in A AND at the same time not in B.
This can also be written as:

$$= P(A \cup B)'$$
$$= 1 - P(A \cup B)$$

$P(A \cup B)$

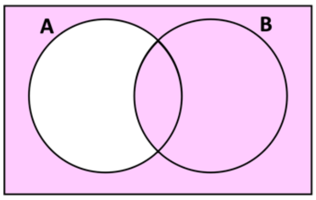


In English: This says everything that is EITHER in A or B.

This is also the same as everything (which has a total of 1) minus the outside part and hence we can also write:

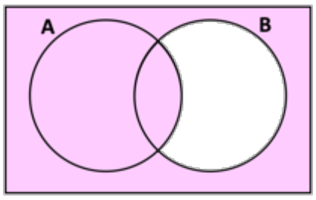
$$= 1 - P(A' \cap B')$$
$$= 1 - P(A \cup B)'$$

$P(A' \cup B)$



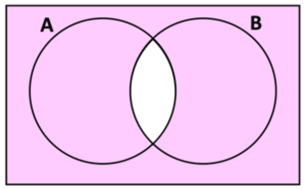
In English: This says everything that is EITHER not in A or in B. Don't let the fact that it says not in A confuse you that we have shaded part of A. The shaded part of A comes from when it could be in B. Remember ∪ means in either. They don't have to be simultaneously like for ∪.

$P(A \cup B')$



In English: This says everything that is EITHER in A or not in B. Don't let the fact that it says not in B confuse you that we have shaded part of B. The shaded part of B comes from when it could be in A. Remember ∪ means in either. They don't have to be simultaneously like for ∪.

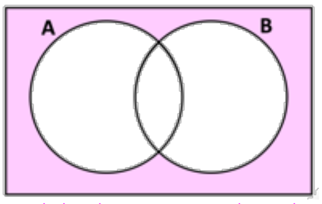
$P(A' \cup B')$



In English: This says everything that is EITHER not in A or not in B. . Don't let the fact that it says not in A and B confuse you that we have shaded part of A and B. The shaded part of B comes from when in not A and the shaded part of A comes when in not B.
This can also be written as:

$$= P(A \cap B)'$$

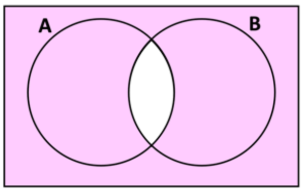
$P(A \cup B)'$



In English: This says everything that is EITHER in A or not in B
This can also be written as:

$$= 1 - P(A \cup B)$$
$$= P(A' \cap B')$$

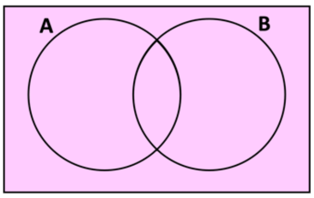
$P(A \cap B)'$



In English: This says consider everything that is in a and B and then shade the reverse
This can also be written as:

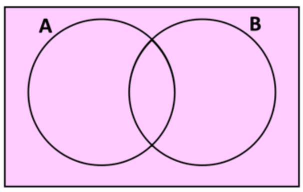
$$= P(A' \cup B')$$

$P(A \cup A')$



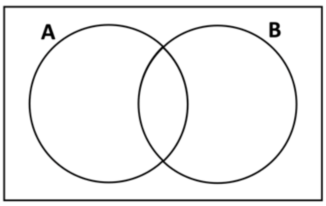
In English: This says everything that is EITHER in A or not in A which is everything

$P(B \cup B')$



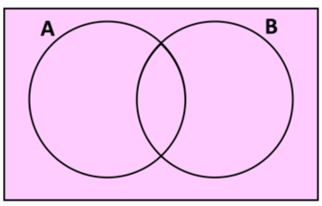
In English: This says everything that is EITHER in B or not in B which is everything

$P(\emptyset)$



In English: This just says the empty set.

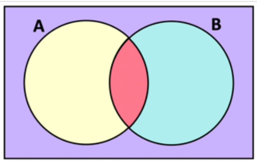
$P(\xi)$



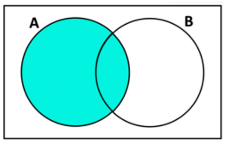
In English: This just says the entire set

Method To Solve Questions:

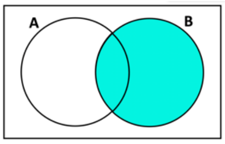
Step 1: Draw out a Venn diagram and fill in as much information as possible from the question. We want to fill in each individual/separate region that we possibly can i.e. we want to know what each colour is in the diagram below.



Let's first see what $P(A)$ and $P(B)$ tell us:



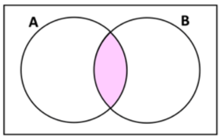
$P(A)$ is allof the circle. This doesn't help us fill out the two separate regions of A though .



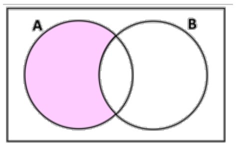
$P(B)$ is all of the circle. his doesn't help us fill out the two separate regions of B though .

We can fill out the following parts thouhh if we are given them:

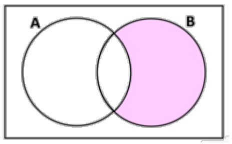
If given $P(A \cap B)$, we can fill in the middle part



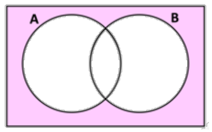
If given $P(A \cap B')$, we can fill in the three quarter moon/crescent of A



If given $P(A' \cap B)$, we can fill in the three quarter moon/crescent of B



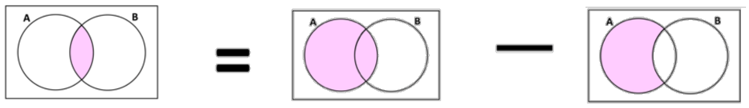
If given $(A' \cap B')$, we can fill in the outside region.

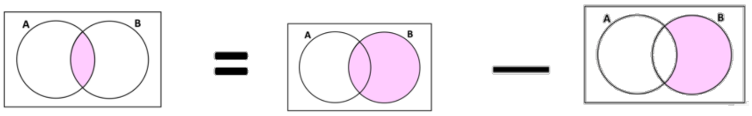


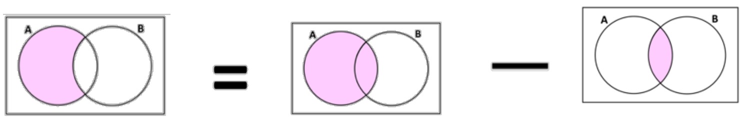
This can also be written as $P(A \cup B)'$

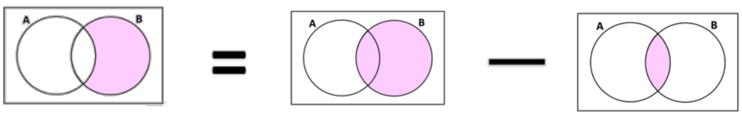
The above are not usually all given and hence not always enough to fill in the whole diagram and answer the question right away (it depends what info we are given in the question). Therefore, we have to take a few steps by using symmetries and/or formula. You can either skip straight to step 2 now and use one of the formulae or try to use one of the following symmetries below (the addiiton formula in step 2 coupled with our information above will deal with this for us so it is not strictly necessary to use the symmetries below, but it will help to solidifiy your understanding of how Venn diagrams work and therefore has been included).

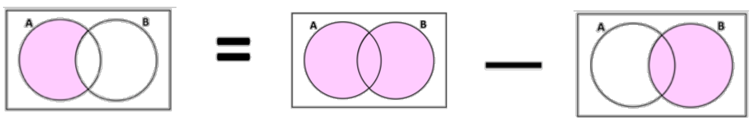
We said above that knowing $P(A)$ and $P(B)$ doesn't help us fill anything in, but it can still help us if we know other information and use symmetry. For example,

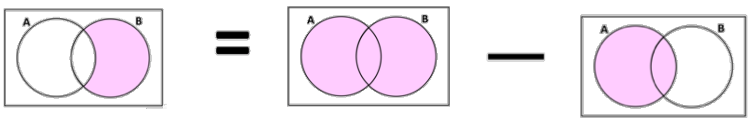


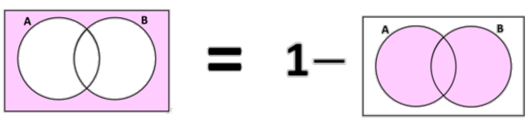










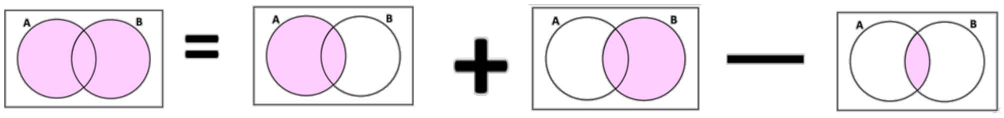


Step 2: Check whether you have enough information to read your answers off from the Venn diagram. If not and you still need more information, use any of the four relevant formulae below.

1) Addition Formula: $\mathbf{P(A \cup B)}$ =P(A)+P(B)$-\mathbf{P(}\boldsymbol{A} \cap \mathbf{B)}$ (most important formula as it is used the most often!)

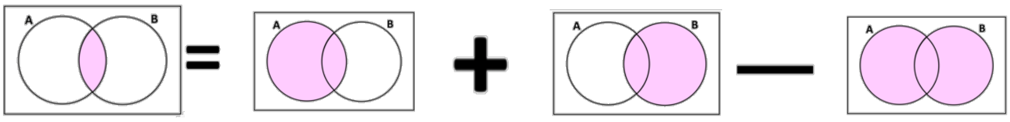
   We use this to find $\mathrm{P(A \cup B)}$ if we have P(A) and P(B) and $\mathrm{P}\ (A \cap \mathrm{B})$

   Visually this rule looks like:



This rule should make sense. When we add A and B we add the middle section twice (double count it) so we must take it away after

We can also re-arrange the formula for $\mathrm{P}\ (A \cap \mathrm{B})$: $P(A \cap \mathrm{B}) =$ P(A)+P(B) $- \mathrm{P}(A \cup \mathrm{B})$



This means we can also use the addition formula to find $\mathrm{P}\ (A \cap \mathrm{B})$ if we have P(A) and P(B) and $\mathrm{P}\ (A \cup \mathrm{B})$

***Important*:** This formula can be adapted for any 2 events (it doesn't have to just be for A and B). For example, for the events A' and B the formula becomes

$$\mathrm{P(A' \cup B)} = \mathrm{P}(A') + \mathrm{P(B)} - \mathrm{P}(A' \cap \mathrm{B})$$

2) Independent: $\mathbf{P(A \cap B) = P(A)P(B)}$
   We can update addition rule to P(A∪B)=P(A)+P(B)$-\mathrm{P(A)P(B)}$ if independent

3) Mutually Exclusive: $\mathbf{P(A \cap B) = 0}$
   We can update addition rule to P(A∪B)=P(A)+P(B) if mutually exclusive

4) Conditional: P(A|$\boldsymbol{B}$) $= \frac{\mathbf{P(A \cap B)}}{\boldsymbol{P(B)}}$
   We can update this formula to $\mathrm{P}(A|B) = P(A)$ if independent

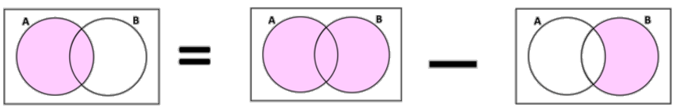
If the questions asks whether independent, use either of the following 2 formulae:

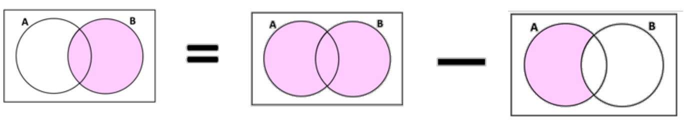
- Find $P(A)$, $P(B)$ and $P(A \cap B)$ and see whether $P(A) \times P(B)$ gives $P(A \cap B)$. This is the most common way.
- Find $\mathrm{P}(A|B)$ and $P(A)$ and see whether they are equal

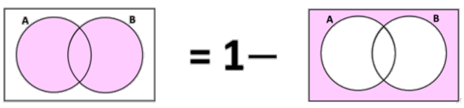
Step 3: Update Venn diagram based on what you've found from using the formulae in step 2

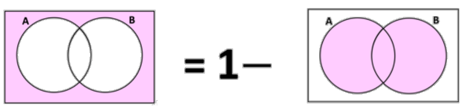
Step 4: Still not enough info to answer the question? Either

- Use the fact that probabilities add to 1
- Use one of the above formulae again
- Use one of the following symmetries









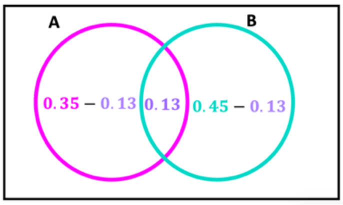
Step 5: Read your answers off of the Venn diagram

Let's look at some examples

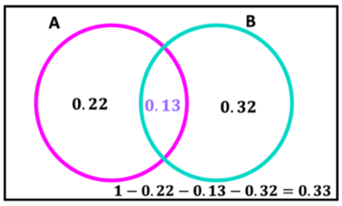
1) P(A)$= 0.35$, P(B)$= 0.45$, P(A∩B)$= 0.13$. Find P(A∪B) and $P(A \cap B')$

Way 1: use a combination of a Venn diagram and formulae (if necessary)

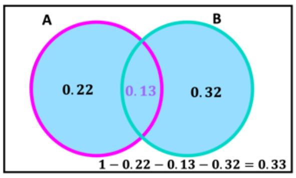
P(A)$= 0.35$
P(B)$= 0.45$
P(A∩B)$= 0.13$



simplify



$P(A \cup B)$ is everything inside the circles

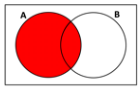


$P(A \cup B) = 0.22 + 0.13 + 0.32 = 0.67$
$P(A \cap B') = 0.22$

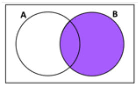
Note: We could have found $P(A \cup B)$ with just using the addition formula from the beginning without any Venn diagram as this question is easy:
Addition formula: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Let's plug in everything we know
$P(A \cup B) = 0.35 + 0.45 - 0.13$
$P(A \cup B) = 0.67$

Way 2: Use the symmetries of a Venn diagram

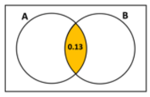
We are told $P(A) = 0.35$. This means the shaded region to the right must represent 0.35



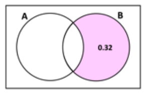
We are told $P(B) = 0.45$. This means the shaded region to the right must represent 0.45



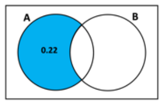
We are told $P(A \cap B)$ =0.13. This means the shaded region to the right must represent 0.13



So, the remaining pink part must be
$0.45 - 0.13 = 0.32$

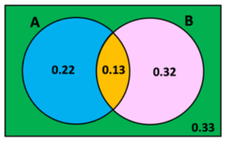


We also can find the remaining blue part
$0.35 - 0.13 = 0.22$



We can now we can fill in the all the pieces of the Venn diagram

$1 - 0.32 - 0.13 - 0.22 = 0.33$



Reading off the diagram:
$P(A \cup B) = 0.22 + 0.13 + 0.32 = 0.67$
$P(A \cap B') = 0.22$

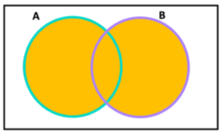
2) P(A)=0.40, P(B)=0.55, P(A∪B)=0.7. Find

i. P($A \cap B$)

ii. P($A \cap B'$)

Way 1: Use a combination of a Venn diagram and formulae

$P(A) = 0.4$, $P(B) = 0.55$ and $P(A \cup B) = 0.7$



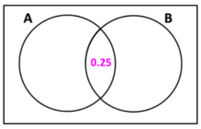
We can't fill into the Venn diagram yet

Let's use the addition formula first to find the intersection: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = 0.40 + 0.55 - P(A \cap B)$

$P(A \cap B) = 0.25$

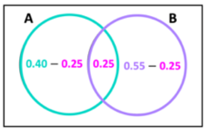
let's update the Venn diagram with $P(A \cap B) = 0.25$



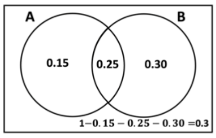
Now we can fill in the rest of the Venn diagram

P(A)=0.40

P(B)=0.55



Let's simplify the numbers

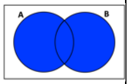


Reading off the diagram:

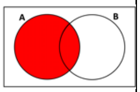
$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

Way 2: Use the symmetries of Venn diagram

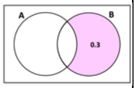
we are told P(A∪B)=0.7. This means the shaded region to the right must represent 0.7



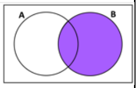
we are also told $P(A) = 0.4$. This means the shaded region to the right must represent 0.4



so, the remaining part must be $0.7 - 0.4 = 0.3$

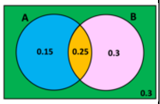


we are also told $P(B) = 0.55$. This means the shaded region to the right must represent 0.55



So, we can now we can fill in the all the pieces of the Venn diagram

$0.55 - 0.3 = 0.25$



$0.4 - 0.25 = 0.15$

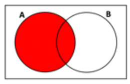
$1 - 0.3 - 0.25 - 0.15 = 0.3$

Reading off the diagram:

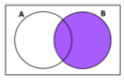
$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

Way 3: Use the symmetries of Venn diagram (same method as way 2, just a different order)

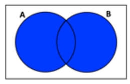
we are also told $P(A) = 0.4$. This means the shaded region to the right must represent 0.4



we are also told $P(B) = 0.55$. This means the shaded region to the right must represent 0.55

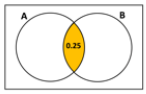


we are told P(A∪B)=0.7. This means the shaded region to the right must represent 0.7



The red and purple added together double counts the middle region. We can find this region by subtracting the red and purple regions from the blue region

$0.7 - 0.40 - 0.55 = 0.25$

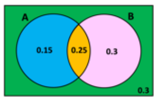


So, we can now we can fill in the all the pieces of the Venn diagram

$0.55 - 0.25 = 0.25$

$0.4 - 0.25 = 0.15$

$1 - 0.3 - 0.25 - 0.15 = 0.3$



Reading off the diagram:

$P(A \cap B) = 0.25$, $P(A \cap B') = 0.15$

A and B are independent if $P(A \cap B) = P(A) \times P(B)$

Let's find each individual part and see whether the above result is true

$$P(A \cap B) = 0.25$$

$$P(A) = 0.40$$

$$P(B) = 0.55$$

$$P(A) \times P(B) = 0.40(0.55) = 0.22$$

$P(A \cap B) \neq P(A) \times P(B)$ since $0.25 \neq 0.22$

Therefore, the events are not independent

3) P(A)=$\frac{11}{36}$, P(B)=$\frac{1}{6}$, $P(A \cup B)'=\frac{21}{36}$. Find

i. P(A∩B)

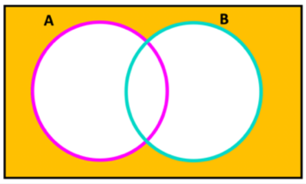
ii. P(A' ∩B)

iii. P($A \cap B'$)

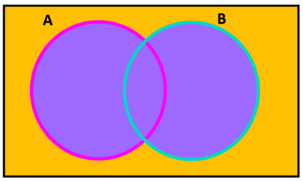
iv. P(A' ∪B)

i.

$P(A) = \frac{11}{36}$, $P(B) = \frac{1}{6}$, $P(A \cup B)' = \frac{21}{36}$



We can't fill into the Venn diagram yet, but we can find out what $P(A \cup B)$ is below by using $P(A \cup B)'$ and the fact that probabilities add to 1



$$P(A \cup B)' = 1 - P(A \cup B)$$

$$P(A \cup B) = 1 - \frac{21}{36} = \frac{15}{36}$$

Let's use the addition formula first to find the intersection

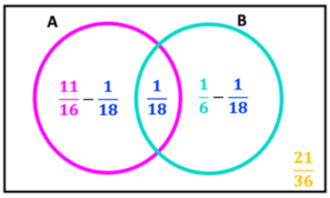
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{15}{36} = \frac{11}{36} + \frac{1}{6} - P(A \cap B)$$

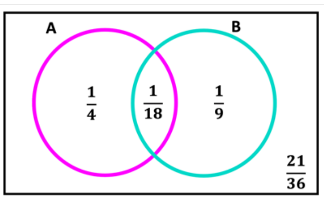
$$P(A \cap B) = \frac{11}{36} + \frac{1}{6} - \frac{15}{36} = \frac{2}{36} = \frac{1}{18}$$

So, we know $P(A) = \frac{11}{36}$, $P(B) = \frac{1}{6}$, $P(A \cup B) = \frac{15}{36}$, $P(A \cap B) = \frac{1}{18}$

we can fill in the Venn diagram now



Simplify



ii. $P(A' \cap B) = \frac{1}{9}$

iii. $P(A \cap B') = \frac{1}{4}$

iv. $P(A' \cup B) = \frac{1}{18} + \frac{1}{9} + \frac{21}{36} = \frac{3}{4}$

Note: Could also have used the addition formula instead and adapted it by replacing A with A'

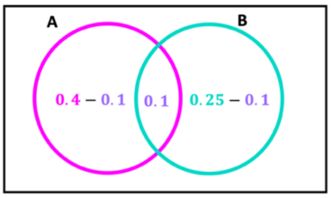
$$P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = \left(\frac{1}{9} + \frac{21}{36}\right) + \left(\frac{1}{18} + \frac{1}{9}\right) - \frac{1}{9} = \frac{3}{4}$$

4) If events A and B are independent and P(A)=0.4 and P(B)=0.25. Find

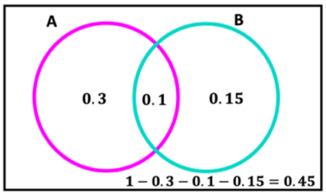
i. P(A∪B)
ii. P(A∩B)
iii. P(A∩B')
iv. P(A'∩B')
v. $P(A \cup B')$

A and B are independent so we can use a formula for this: $P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.25 = 0.1$

So, we have
P(A)=0.4
P(B)=0.25
$P(A \cap B) = 0.1$



simplify



Way 1: Read off Venn Diagram

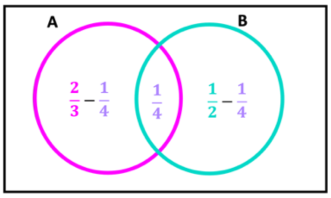
i. $P(A \cup B)$ is the inside of the circles
$P(A \cup B) = 0.3 + 0.1 + 0.1 = 0.55$
ii. $P(A \cap B)$ is the middle part common to both circles
$P(A \cap B)$ =0.1
iii. $P(A \cap B')$ means in A, but not in B, so this is the half moon/crescent on the left
$P(A \cap B') = 0.3$
iv. $P(A' \cap B')$ is the outside region
$P(A' \cap B') = 0.45$
v. $P(A \cup B') = 0.3 + 0.1 + 0.45 = 0.85$
Note: Could also have used the addition formula instead and adapted it by replacing B with B'
$P(A \cup B') = P(A) + P(B') - P(A \cap B') = (0.3 + 0.1) + (0.3 + 0.45) - 0.3 = 0.85$

Way 2: Use Formulae
i. $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.25 - 0.1 = 0.55$
ii. $P(A \cap B) = P(A) \times P(B) = 0.4 \times 0.25 = 0.1$
Note: we could only multiply since the events are independent
iii. $P(A \cap B') = P(A) \times \left(1 - P(B)\right) = 0.4 \times (1 - 0.25) = 0.4 \times 0.75 = 0.3$
Note: we could only multiply since the events are independent
iv. $P(A' \cap B') = \left(1 - P(A)\right) \times (1 - P(B)) = (1 - 0.4) \times (1 - 0.25) = 0.6 \times 0.75 = 0.45$
Note: we could only multiply since the events are independent
v. $P(A \cup B') = P(A) + P(B') - P(A \cap B') = 0.4 + 0.75 - 0.3 = 0.85$

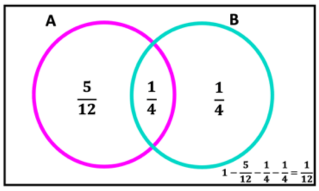
5) P(A)= $\frac{2}{3}$, P(B)= $\frac{1}{2}$, P(A∩B)= $\frac{1}{4}$. Find P(A∪B) P(A∩B)' and P(B|A)

Way 1: Use a combination of Venn diagram and formulae

P(A)= $\frac{2}{3}$
P(B)= $\frac{1}{2}$
P(A∩B)= $\frac{1}{4}$



simplify



$P(A \cup B)$ is the inside of the circles

$$P(A \cup B) = \frac{5}{12} + \frac{1}{4} + \frac{1}{4} = \frac{11}{12}$$

$P(A \cap B)'$ is the outside region

$$P(A \cap B)' = \frac{1}{12}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

Way 2: Use formulae only

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{2}{3} + \frac{1}{2} - \frac{1}{4} = \frac{11}{12}$$

$$P(A \cap B)' = 1 - P(A \cap B) = 1 - \frac{11}{12} = \frac{1}{12}$$

$P(A|B)$ has a formula

$$\frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{4}}{\frac{2}{3}} = \frac{3}{8}$$

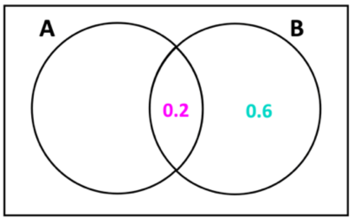
6) Events A and B are independent with $P(A \cap B) = 0.2$ and $P(A' \cap B) = 0.6$

i. Find $P(B)$

ii. Find $P(A \cup B)$

Way 1: Use a combination of Venn diagram and formulae

P(A∩ B)=0.2
P(A'∩ B)=0.6



i.

$$P(B) = 0.2 + 0.6 = 0.8$$

ii.

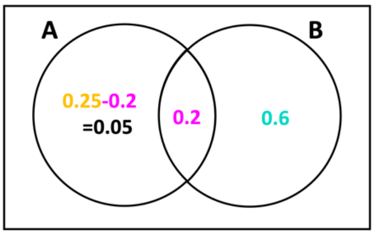
Given A and B Independent:

$P(A \cap B) = P(A) \times P(B)$

$0.2 = P(A) \times (0.6 + 0.2)$

$P(A) = \frac{0.2}{0.8} = 0.25$

Let's update the Venn diagram



$P(A \cup B) = 0.05 + 0.2 + 0.6 = 0.85$

Way 2: use formulae only

i. $P(B) = P(A \cap B) + P(A' \cap B) = 0.2 + 0.6 = 0.8$

ii. A and B are independent so $P(A \cap B) = P(A) \times P(B)$

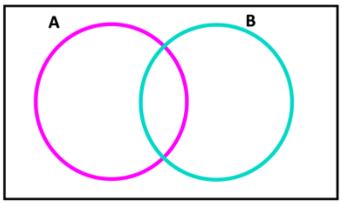
$$\text{so } 0.2 = P(A) \times 0.8$$
$$P(A) = \frac{0.2}{0.8} = 0.25$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.25 + 0.8 - 0.2 = 0.85$$

7) P(A)= $\frac{2}{5}$, P(B)= $\frac{11}{20}$, P(A|B)= $\frac{2}{11}$. Find P(A∩B), P(A∪B). Are these events independent?

Way 1: Use a combination of Venn diagram and formulae

P(A)= $\frac{2}{5}$, P(B)= $\frac{11}{20}$



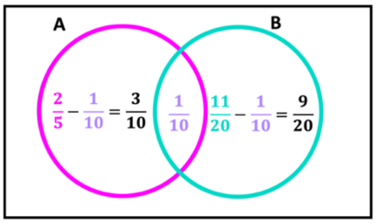
We don't have enough to fill into the Venn diagram yet

Given P(A|B)= $\frac{2}{11}$

We can use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{P(A\cap B)}{\frac{11}{20}} = \frac{2}{11}$$

$$P(A\cap B) = \frac{2}{11} \times \frac{11}{20} = \frac{1}{10}$$



$$P(A\cup B) = \frac{3}{10} + \frac{1}{10} + \frac{9}{20} = \frac{17}{20}$$

Way 2: Use formulae only

Given P(A|B)= $\frac{2}{11}$

Let's use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{2}{11} = \frac{P(A\cap B)}{\frac{11}{20}}$$

$$P(A\cap B) = \frac{2}{11} \times \frac{11}{20} = \frac{1}{10}$$

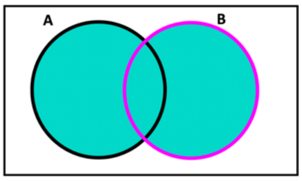
$$P(A\cup B) = P(A) + P(B) - P(A\cap B) = \frac{2}{5} + \frac{11}{20} - \frac{1}{10} = \frac{17}{20}$$

$P(A|B) = \frac{2}{11}$ but $P(A) = \frac{2}{5}\frac{2}{11} \neq \frac{2}{5}$ so A and B are **not** independent.

8) P(B)= $\frac{1}{2}$, P(A∪ B)= $\frac{13}{20}$, P(A|B)= $\frac{2}{5}$. Find P(A∩B), P(A), P(B|A), P(A'∩ B)

Way 1: Use a combination of Venn diagram and formulae

P(B)= $\frac{1}{2}$, P(A∪ B)= $\frac{13}{20}$



We don't have enough to fill into the Venn diagram yet

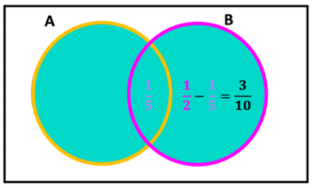
Given P(A|B)= $\frac{2}{5}$

We can use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{P(A\cap B)}{\frac{1}{2}} = \frac{2}{5}$$

$$P(A\cap B) = \frac{2}{5}\times\frac{1}{2} = \frac{1}{5}$$

We can now fill in the middle part



P(A)= $\frac{13}{20} - \frac{3}{10} = \frac{7}{20}$

Let's use the conditional formula to find $P(B|A)$

$$P(B|A) = \frac{P(B\cap A)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

P(A'∩ B) $= \frac{3}{10}$

Way 2: Use formulae only

Given P(A|B)= $\frac{2}{5}$

Let's use the conditional formula $P(A|B) = \frac{P(A\cap B)}{P(B)}$

$$\frac{2}{5} = \frac{P(A\cap B)}{\frac{1}{2}}$$

$$P(A\cap B) = \frac{2}{5}\times\frac{1}{2} = \frac{1}{5}$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$\frac{13}{20} = P(A) + \frac{1}{2} - \frac{1}{5}$$

$$P(A) = \frac{13}{20} + \frac{1}{5} - \frac{1}{2} = \frac{7}{20}$$

$$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

$$P(B) = P(A\cap B) + P(A'\cap B)$$

$$\frac{1}{2} = \frac{1}{5} + P(A'\cap B)$$

$$P(A'\cap B) = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

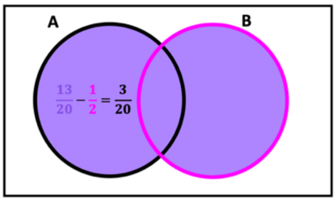
9) $P(B)=\frac{1}{2}$, $P(A|B)=\frac{2}{5}$, $P(A\cup B)=\frac{13}{20}$. Find $P(A\cap B)$, $P(A'\cap B)$, $P(A)$, $P(A|B')$, $P(B|A)$, and $P(A'\cup B)$

Way 1: Use a combination of Venn diagram and formulae

$P(B)=\frac{1}{2}$

$P(A|B)=\frac{2}{5}$

$P(A\cup B)=\frac{13}{20}$



We don't have enough to fill in the whole Venn diagram yet

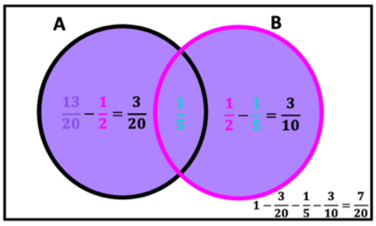
Let's use the conditional formula

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$\frac{2}{5}=\frac{P(A\cap B)}{\frac{1}{2}}$$

$$P(A\cap B)=\frac{2}{5}\times\frac{1}{2}=\frac{1}{5}$$

Now we can fill in the rest of the Venn diagram



$$P(A)=\frac{3}{20}+\frac{1}{5}=\frac{7}{20}$$

$$P(A|B')=\frac{P(A\cap B')}{P(B')}=\frac{\frac{3}{20}}{\frac{3}{20}+\frac{7}{20}}=\frac{3}{10}$$

$$P(B|A)=\frac{P(A\cap B)}{P(A)}=\frac{\frac{1}{5}}{\frac{7}{20}}=\frac{4}{7}$$

$$P(A'\cup B)=\frac{3}{10}+\frac{7}{20}+\frac{1}{5}=\frac{17}{20}$$

Note: Could also have used the addition formula instead and adapted it by replacing A with A'

$$P(A'\cup B)=P(A')+P(B)-P(A'\cap B)=\left(\frac{3}{10}+\frac{7}{20}\right)+\left(\frac{1}{5}+\frac{3}{10}\right)-\frac{3}{10}=\frac{17}{20}$$

Way 2: Use formulae only (harder)

Conditional formula:

$$P(A|B)=\frac{P(A\cap B)}{P(B)}$$

$$\frac{2}{5}=\frac{P(A\cap B)}{\frac{1}{2}}$$

Independence formula:

$$P(A\cap B)=\frac{2}{5}\times\frac{1}{2}=\frac{1}{5}$$

We can find P(B)

$$P(B)=P(A\cap B)+P(A'\cap B)$$

$$\frac{1}{2} = \frac{1}{5} + P(A' \cap B)$$
$$P(A' \cap B) = \frac{1}{2} - \frac{1}{5} = \frac{3}{10}$$

Additional formula:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$\frac{13}{20} = P(A) + \frac{1}{2} - \frac{1}{5}$$
$$P(A) = \frac{13}{20} - \frac{1}{2} + \frac{1}{5} = \frac{7}{20}$$

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{P(A) - P(A \cap B)}{1 - P(B)} = \frac{\frac{7}{20} - \frac{1}{5}}{1 - \frac{1}{2}} = \frac{\frac{3}{20}}{\frac{1}{2}} = \frac{3}{10}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{1}{5}}{\frac{7}{20}} = \frac{4}{7}$$

$$P(A' \cup B) = P(A') + P(B) - P(A' \cap B) = \left(1 - P(A)\right) + P(B) - P(A' \cap B)$$
$$= \left(1 - \frac{7}{20}\right) + \frac{1}{2} - \frac{3}{10} = \frac{13}{20} + \frac{1}{2} - \frac{3}{10} = \frac{17}{20}$$

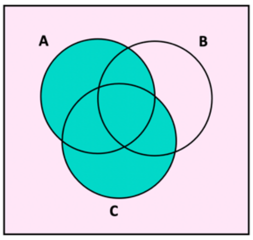
What about if we have 3 events?

Let's look at 3 examples and apply what we already know:

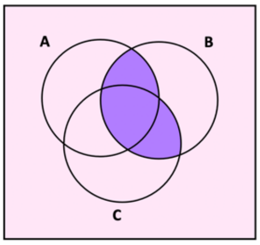
<table>
<tr><th>P ($(A \cup C) \cap B$)</th><th>P ($B' \cap (A \cap C)$)</th><th>P ($(A \cap B) \cup C$)</th></tr>
<tr>
<td>

$P((A \cup C) \cap B)$

We deal with what is in the bracket first. This means we shade $P(A \cup C)$ first



Now we deal with the $\cap B$ part. This means we choose the parts that are also in B i.e. remove anything that is not in B



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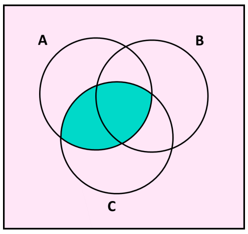
<td>

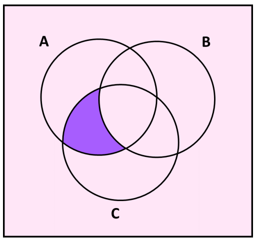
$P(B' \cap (A \cap C))$

We deal with what is in the bracket first. So we shade $P(A \cap C)$ first



Now we deal with the $B' \cap$ part

This means we are choosing the parts of $A \cap C$ that are also NOT in B, ie. remove the bits that are in B



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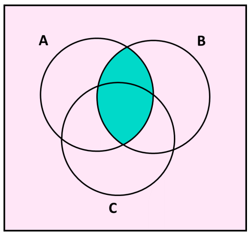
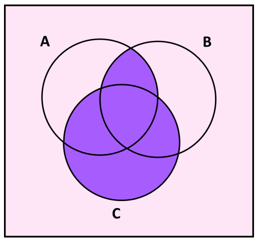
<td>

$P((A \cap B) \cup C)$

consider the bracket first; shade $A \cap B$



Now deal with $\cup C$ so also shade everything in C



</td>
</tr>
</table>

1) A, B and C are three events with $P(A) = 0.24, P(B) = 0.4, P(C) = 0.45, P(A \cap B \cap C) = 0.1$. Given that A and B are independent, B and C are independent and $A \cap B' \cap C = \emptyset$

i. Draw a Venn diagram to illustrate the probabilities

Find:

ii. $P(A \cap (B' \cup C))$

iii. $P((A \cup B) \cap C)$

iv. State, with reasons, whether events A' and C are independent

---

i.

$P(A) = 0.25$

$P(B) = 0.4$

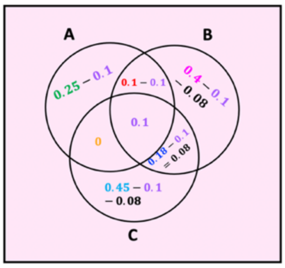
$P(C) = 0.45$

$P(A \cap B \cap C) = 0.1$

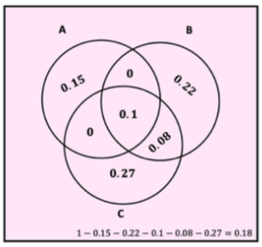
$A \cap B' \cap C = \emptyset$

$P(A \cap B) = P(A)P(B) = 0.25(0.4) = 0.1$

$P(B \cap C) = P(B)P(C) = 0.4(0.45) = 0.18$



We can simplify the numbers

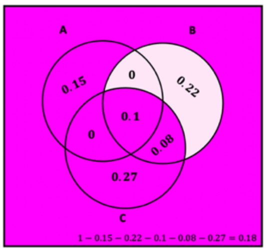


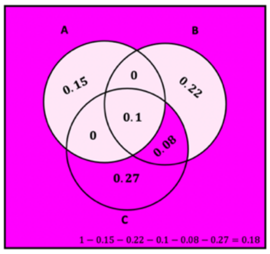
ii.

$P(A' \cap (B' \cup C))$

Shade $P(B' \cup C)$ first – left diagram

Now choose the parts that are also NOT in A – right diagram



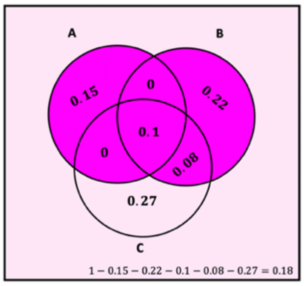


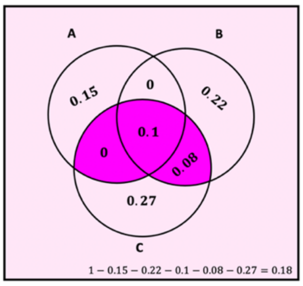
$$0.08 + 0.27 + 0.18 = 0.53$$

iii.

$$P(A \cup B) \cap C)$$

Shade $P(A \cup B)$ first – left diagram
Now choose the bits that are also in C – right diagram

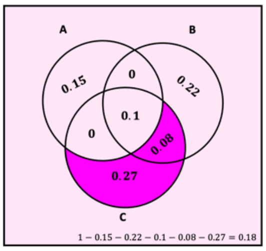




$$0 + 0.1 + 0.08 = 0.18$$

iv.

If independent $P(A' \cap C) = P(A')P(C)$
$P(A') = 1 - 0.25 = 0.75$ (since A was given in question)
$P(C) = 0.45$ (since C was given in question)



$$P(A' \cap C) = 0.27 + 0.08 = 0.35$$
$$P(A')P(C) = 0.75(0.45) = 0.3375$$
$$0.35 \neq 0.3375$$

So not independent

2) A, B and C are three events with $P(A) = 0.55, P(B) = 0.35, P(C) = 0.4, P(A \cap C) = 0.2.$ Given that A and B are mutually exclusive, and B and C are independent

i. Draw a Venn diagram to illustrate the probabilities

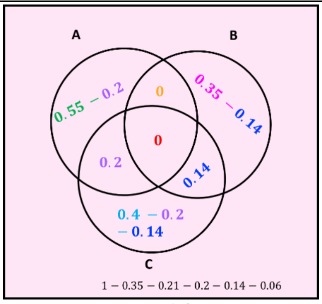
Find

v. $P(A' \cap B')$
vi. $P\big(A \cup (B \cap C')\big)$
vii. $P(A \cap C)' \cup B)$

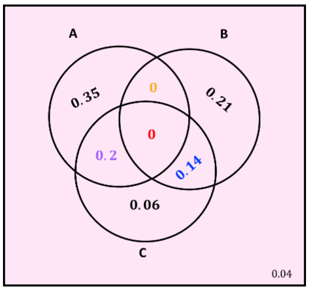
$$P(A) = 0.55$$
$$P(B) = 0.35$$
$$P(C) = 0.4$$
$$P(A \cap C) = 0.2$$
$$P(A \cap B) = 0$$
$$P(B \cap C) = P(B)P(C) = 0.35(0.4) = 0.14$$

A and B are mutually exclusive so $P(A \cap B) = 0$

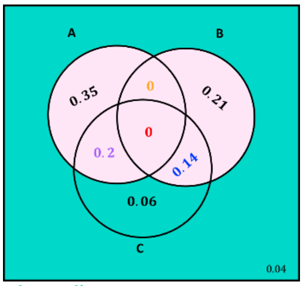
i.



Simplify

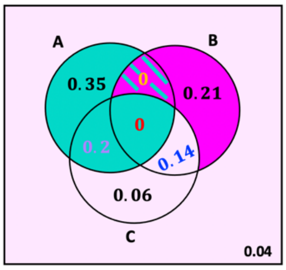


ii.



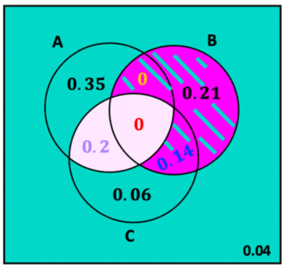
$P(A' \cap B') = 0.06 + 0.04 = 0.1$

iii.



$P(A \cup (B \cap C')) = 0.35 + 0.2 + 0.21 = 0.76$

iv.

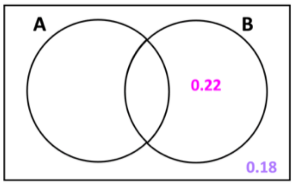


iv. $P((A \cap C)' \cup B) = 0.35 + 0.21 + 0.14 + 0.06 + 0.04 = 0.8$

The last thing we have to watch out for is Venn diagram questions with algebra. Don't be afraid to call an unknown x in the Venn diagram or formula and build your own equations.

1) $P(A' \cap B) = 0.22, P(A' \cap B') = 0.18. P(AIB) = 0.6.$
   i. Find P(A)
   ii. P(A∪B)
   iii. P(A∩B)
   iv. Are A and B independent?

$P(A' \cap B) = 0.22$
$P(A' \cap B') = 0.18$



There isn't much that can be filled into the Venn diagram

Let's use: P(AIB)=0.6

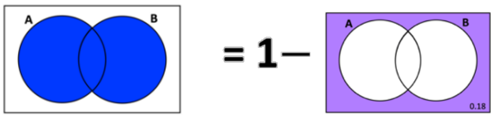
This means $\frac{P(A\cap B)}{P(B)} = 0.6$

This doesn't help either

This is hard since can't fill into the formula in order fill out the rest of hte Venn diagram. So, we must use the symmetries below instead.

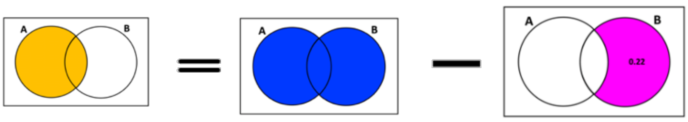
i. and ii.

Way 1:



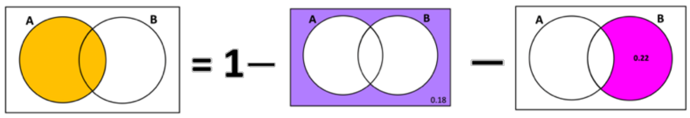
So $P(A \cup B) = 1 - 0.18 = 0.82$

Now we can find $P(A)$ since

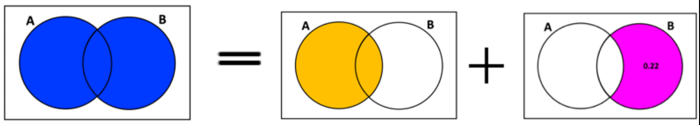


Find $P(A) = 0.82 - 0.22 = 0.6$

Way 2:



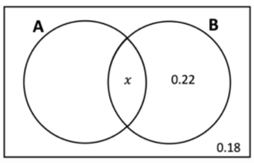
$P(A) = 1 - 0.18 - 0.22 = 0.6$



$P(A \cup B) = 0.6 + 0.22 = 0.82$

iii.

We still don't have enough info to fill all numbers into the Venn diagram. To get around this, lets we call the intersection part x



P(AIB)=0.6

$$\frac{x}{x + 0.22} = 0.6$$
$$x = 0.33$$

Note: Or you can use that $P(A'|B) = \frac{P(A' \cap B)}{P(B)}$

$$1 - 0.6 = \frac{0.22}{P(B)}$$
$$P(B) = 0.55$$
$$P(A \cap B) = 0.55 - 0.33 = 0.22$$

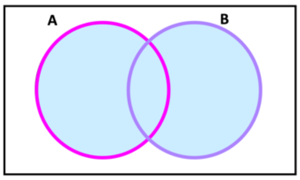
iv.

$P(A) = 0.6, P(B) = 0.22 + 0.33 = 0.55$
$P(A \cap B) = 0.33$
$0.33 = 0.6(0.55)$ therefore independent

2) P(A)=$\frac{1}{4}$, $P(A \cup B) = \frac{2}{3}$. A and B are independent. Find P(B), P(A′∩B) P(B′|A)

P(A) $= \frac{1}{4}$

$P(A \cup B) = \frac{2}{3}$



Not enough into to fill in the Venn diagram

A and B are independent: we can use the formula $P(A \cap B) = P(A)P(B)$
This doesn't help since we don't have enough info to fill in

We can also use the addition formula:
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$ can be updated as
$P(A \cup B) = P(A) + P(B) - P(A)P(B)$ since independent
$\frac{2}{3} = \frac{1}{4} + P(B) - \frac{1}{4}P(B)$
Solve for $P(B)$. Call if x if it helps look a bit more familiar to the types of equations you're used to solving.

$$\frac{2}{3} - \frac{1}{4} = x - \frac{1}{4}x$$

$$\frac{5}{12} = \frac{3}{4}x$$

$$x = \frac{\frac{5}{12}}{\frac{3}{4}} = \frac{5}{9}$$

So $P(B) = x = \frac{5}{9}$

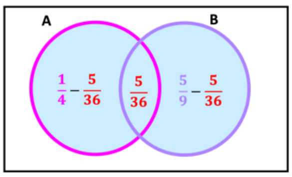
Now we have enough info to fill into the Venn diagram

P(A)= $\frac{1}{4}$

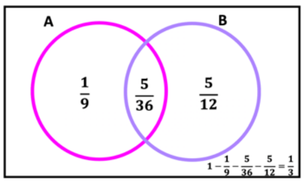
$P(A \cup B) = \frac{2}{3}$

$P(B) = \frac{5}{9}$

A and B are independent so $P(A \cap B) = P(A) \times P(B) = \frac{1}{4} \times \frac{5}{9} = \frac{5}{36}$



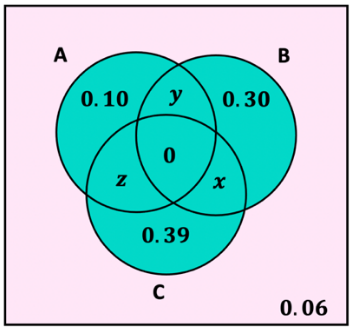
Simplify



From Venn Diagram, $P(A' \cap B) = \frac{5}{12}$

$$P(B'|A) = \frac{P(B' \cap A)}{P(A)} = \frac{\frac{1}{9}}{\frac{1}{9} + \frac{5}{36}} = \frac{\frac{1}{9}}{\frac{1}{4}} = \frac{4}{9}$$

3) The Venn diagram shows 3 events, A, B and C, and their associated probabilities. Events B and C are mutually exclusive. Events A and C are independent. Showing your working, find the value of x, the value of y and the value of z.



We can form 3 equations and have 3 unknowns so we can find each unknown

A and B mutually exclusive: $x + 0 = 0$
A and C independent: $z = (0.10 + y + z)(z + x + 0.39)$
Probabilities add to make 1: $0.1 + z + y + 0.3 + x + 0.39 + 0.06 = 1$

$$z + y + 0.85 = 1$$
$$z + y = 0.15$$

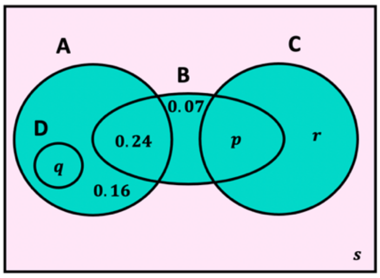
$z = (0.10 + y + z)(z + x + 0.39)$ becomes

$$z = (0.10 + 0.15)(z + 0 + 0.39)$$
$$z = 0.25(z + 0.39)$$
$$z = 0.25z + 0.0975$$
$$0.75z = 0.0975$$
$$z = 0.13$$

$$0.13 + y = 0.15$$
$$y = 0.02$$

$$x = 0, y = 0.02, z = 0.13$$

4) The Venn diagram shows the probabilities associated with four events, A, B, C and D



i. Write down any pair of mutually exclusive events from A, B, C and D

Given that P(B) = 0.4

ii. find the value of p

Given also that A and B are independent

iii. find the value of q

Given further that P (B′ | C) = 0.64, find

iv. the value of r

v. the value of s

i.

Pairs of mutually exclusive events are where there is no overlap on the Venn diagram:

A and C
B and D
C and D

ii.

$$\mathbf{P(B) = 0.4}$$
$$p + 0.07 + 0.24 = 0.4$$
$$p = 0.09$$

iii.

$$\boldsymbol{P(A \cap B) = P(A) \times P(B)}$$
$$(0.24 + 0,.16 + q)0.4 = 0.24$$
$$0.24 + 0.16 + q = 0.6$$
$$q = 0.2$$

iv.

$$\boldsymbol{P\,(\,B'\,|\,C\,)\ =\ 0.64}:$$
$$\frac{P(B' \cap C)}{P(C)} = 0.64$$
$$\frac{r}{p + r} = 0.64$$
$$\frac{r}{0.09 + r} = 0.64$$
$$r = 0.64(0.09 + r)$$
$$r = 0.0576 + 0.64r$$
$$r - 0.64r = 0.0576$$
$$0.36r = 0.0576$$
$$r = 0.16$$

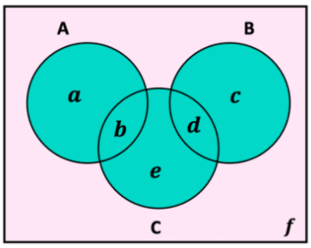
v. Probabilities add to 1

$$0.16 + 0.24 + 0.2 + 0.07 + 0.09 + 0.16 + s = 1$$
$$s = 1 - 0.92 = 0.08$$

5) Three events A, B and C are such that A and B are mutually exclusive and P(A)=0.2, P(C)=0.3 and P(A∪ $B) = 0.4$ and P(B∪ C)=0.34

i. Calculate P(B) and P(B∩ C)

ii. Determine whether B and C are independent



i. We can form equations based on all the given info

We can form 5 equations, but have 6 unknowns so we could never find each unknown, but that's ok

P(A)=0.2 : $a + b = 0.2$

P(C)=0.3 : $b + d + e = 0.3$

P(A∪ $\boldsymbol{B}) = \mathbf{0.4}$: $a + b + c + d = 0.4$

$$a + b + c + d = 0.4$$
$$0.2 + c + d = 0.4$$
$$c + d = 0.2$$

P(B∪ $\boldsymbol{C}$)=0.34 : $b + c + d + e = 0.34$

$$b + c + d + e = 0.34$$
$$0.3 + c = 0.34$$
$$c = 0.04$$

So $c + d = 0.2$ becomes $0.04 + d = 0.2 \Rightarrow d = 0.16$

Probabilities add the 1: $a + b + c + d + e + f = 1$ (we don't need this)

$$P(B) = c + d = 0.2$$

$$P(B \cap C) = d = 0.16$$

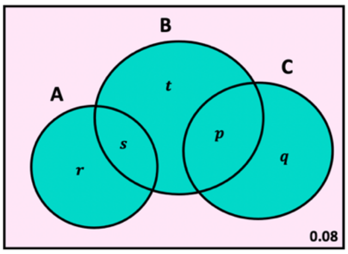
ii. Are B and C independent?

$$P(B) = 0.2$$
$$P(C) = 0.3$$
$$P(B \cap C) = 0.16$$
$$(0.2)(0.3) = 0.06$$
$$0.06 \neq 0.16$$

Therefore $P(A \cap B) \neq P(A)P(B)$

not independent

6) The Venn diagram shows three events, A,B and C where $p, q, r, s, and\ t$ are probabilities. P(A)=0.5, P(B)=0.6 and P(C)=0.25 and the events B and C are independent



i. Find the value of p and the value of q
ii. Find the value of r
iii. Hence write down the value of s and the value of t
iv. State, giving a reason, whether or not the events A and B are independent
v. Find P(B| AUC)

i. We can form equations based on all the given info

We can form 5 equations and 5 unknowns so we can find each unknown

P(A)=0.5 : $r + s = 0.5$

P(B)=0.6 : $s + t + p = 0.6$

P(C) $= \mathbf{0.25}$: $p + q = 0.25$

Events B and C are independent : $(s + t + p)(p + q) = p$

$$(0.6)(0.25) = p$$
$$p = 0.15$$

So $p + q = 0.25$ becomes

$$0.15 + q = 0.25$$
$$q = 0.1$$

ii. **Probabilities add the 1:** $r + s + t + p + 0.1 + 0.08 = 1$

$$r + s + t + p + 0.1 + 0.08 = 1$$
$$r + 0.6 + 0.1 + 0.08 = 1$$
$$r = 1 - 0.6 - 0.1 - 0.08$$
$$r = 0.22$$

iii. $r + s = 0.5$ becomes

$$0.22 + s = 0.5$$
$$s = 0.5 - 0.22 = 0.28$$

$s + t + p = 0.6$ becomes

$$0.28 + t + 0.15 = 0.6$$
$$t = 0.6 - 0.28 - 0.15$$

$$t = 0.17$$

iv. $P(A) = 0.5$ (given in question)
$P(B) = 0.6$ (given in question)
$P(A \cap B) = s = 0.28$
$(0.5)(0.6) = 0.30$
$0.28 \neq 0.30$
Therefore $P(A \cap B) \neq P(A)P(B)$
Not independent

$$\mathrm{P(B \mid A \cup C)} = \frac{P(B \cap A \cup C)}{P(A \cup C)} = \frac{s+p}{r+s+p+q} = \frac{0.28+0.15}{0.5+0.25} = 0.573$$

7) Each of the 30 students in a class plays at least one of squash, hockey and tennis
18 students play squash
19 students play hockey
17 students play tennis
8 students play squash and hockey
9 students play hockey and tennis
11 students play squash and tennis

i. Find the number of students who play all three sports

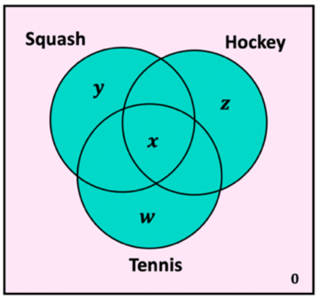
A student is picked at random from the class

ii. Given that this student plays squash, find the probability that this student does not play hockey

Two different students are picked at random from the class, one after the other, without replacement.

iii. Given that the first student plays squash, find the probability that the second student plays hockey

Way 1



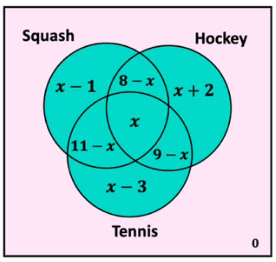
Let's find w, x, y and z in terms of x

We know the following

$$y + 11 + (8 - x) = 18$$
$$y = x - 1$$

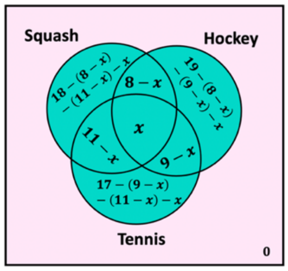
$$w + 11 + (9 - x) = 17$$
$$w = x - 3$$

$$z + 8 + (9 - x) = 19$$
$$z = x + 2$$

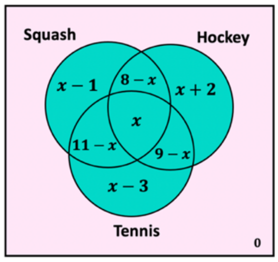


The total of these is equal to 30
$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$
$$x + 26 = 30$$

Way 2



Let's simplify each part by collecting like terms



The total of these is equal to 30
$$x - 1 + 8 - x + x + 2 + x + 11 - x + 9 - x + x - 3 = 30$$
$$x + 26 = 30$$
$$x = 4$$

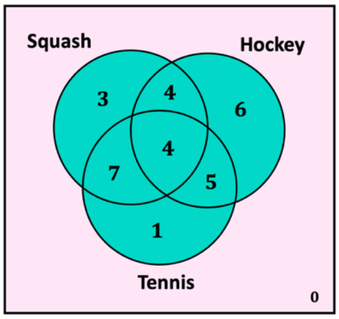
Hence the number of people who play all three sports is 4

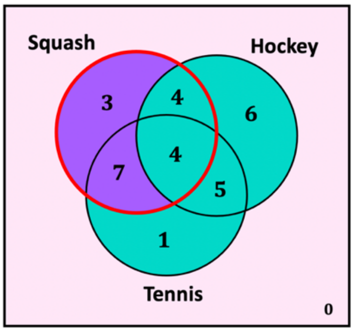
$$x = 4$$

Hence the number of people who play all three sports is 4

ii.

Now that we know x we can fill out the rest of the diagram



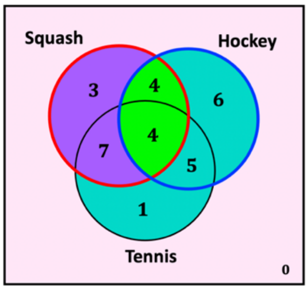


$$P(H'|S) = \frac{P(H' \cap S)}{P(S)} = \frac{7+3}{7+3+4+4} = \frac{10}{18} = \frac{5}{9}$$

i. Think back to when you first learnt probability and you were taking beads out of a bag without replacement. What happens in the first selection affected the numbers for what happens in the next selection. We care whether the first student played hockey or not as this will affect the numbers when selecting for the hockey. So, it should make sense that there are 2 cases you need to consider here. Either the first person plays hocket as well as squash, or the first person doesn't don't play hockey.

$$P(H|S) \times P(H) + P(H'|S\,) \times P(H)$$

Where the first even in each is the first student and the second event in each is the second student



choose all the options out of the red Spanish circle for the first case, but we take them in cases (green and purple) since they affect Hockey differently

$$\left(\frac{4+4}{18}\right)\left(\frac{18}{29}\right) + \left(\frac{7+3}{18}\right)\left(\frac{19}{29}\right) = \frac{167}{261}$$