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Definitions And Formulae

Consider the following table of frequencies

Time (mins)	Frequency (f)
$0 < t \leq 5$	2
$5 < t \leq 10$	4
$10 < t \leq 20$	16
$20 < t \leq 30$	8
$30 < t \leq 60$	5

For example:
The 2 tells us that 2 people took between 0 and 5 minutes
The 4 tells us that 4 people took between 5 and 10 minutes
The 16 tells us that 16 people took between 10 and 20 minutes
The 8 tells us that 8 people took between 20 and 30 minutes
The 5 tells us that 5 people took between 30 and 60 minutes

If given the table above of frequencies, we can find the class width (cw) and frequency density (fd). Let's first look at class widths. Class width is the difference between the upper-class limit and the lower-class limit of a class interval.

Time (mins)	Frequency (f)	Class width
$0 < t \leq 5$	2	$5 - 0 = 5$
$5 < t \leq 10$	4	$10 - 5 = 5$
$10 < t \leq 20$	16	$20 - 10 = 10$
$20 < t \leq 30$	8	$30 - 20 = 10$
$30 < t \leq 60$	5	$60 - 30 = 30$

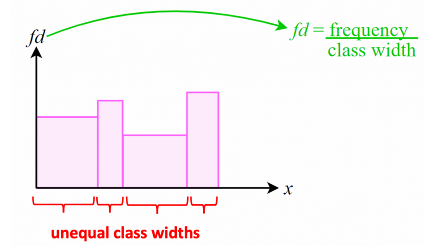
For example:
The 5 tells us that the length of the bar for $0 < t \leq 5$ is 5
The 5 tells us that the length of the bar for $5 < t \leq 10$ is 5
The 10 tells us that the length of the bar for $10 < t \leq 20$ is 10
The 10 tells us that the length of the bar for $20 < t \leq 30$ is 10
The 30 tells us that the length of the bar for $30 < t \leq 60$ is 30

Histograms are like bar charts, but they can have different widths as you can see above (some are 5, some are 10 and one is 30).
We use frequency density with histograms to allow for a meaningful comparison of different classes where the class widths may not be equal. If the class widths were the same (which they barely ever are) then we would just use the frequency and not even bother to find the frequency density. The frequency density is the frequency divided by the class width.

Time (mins)	Frequency (f)	Class width	Frequency Density
$0 < t \leq 5$	2	$5 - 0 = 5$	$\frac{2}{5} = 0.4$
$5 < t \leq 10$	4	$10 - 5 = 5$	$\frac{4}{5} = 0.8$
$10 < t \leq 20$	16	$20 - 10 = 10$	$\frac{16}{10} = 1.6$
$20 < t \leq 30$	8	$30 - 20 = 10$	$\frac{8}{10} = 0.8$
$30 < t \leq 60$	5	$60 - 30 = 30$	$\frac{5}{30} = 0.17$

How To Draw A Histogram

We plot the frequency density (fd) on the y axis and the bars with the class widths on the x axis. This looks just like a bar chart but notice how there are **no gaps between** bars, unlike with bar charts.



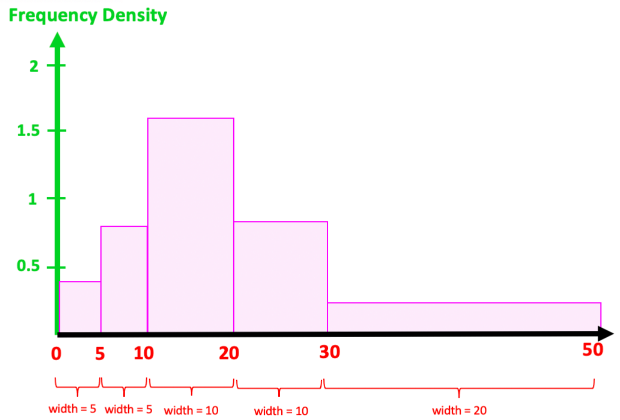
For example:

Time (mins)	Frequency (f)
$0 < t \leq 5$	2
$5 < t \leq 10$	4
$10 < t \leq 20$	16
$20 < t \leq 30$	8
$30 < t \leq 60$	5

we find $\Longrightarrow$

Class width	Frequency Density
5	$\frac{2}{5} = 0.4$
5	$\frac{4}{5} = 0.8$
10	$\frac{16}{10} = 1.6$
10	$\frac{8}{10} = 0.8$
30	$\frac{5}{30} = 0.17$

which gives the graph $\Longrightarrow$



notice how there are no gaps between each bar

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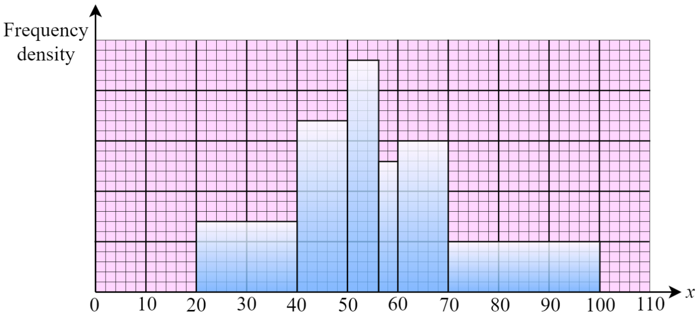
When Do We Use Histograms To Represent Our Data?

We use histograms when our data is continuous! What is continuous data? Discrete data is counted and continuous data is measured.
Discrete Data can only take certain values (such as 1,2,3,4,5), whereas continuous Data can take any value within a range.
Examples of discrete data include shoe size, number of students present, score in basketball game, number of fish in lake.
Examples of continuous data include time, volume of water in lake, height, weight and distance.

Calculations

We are often give a table but with gaps in it (some unknown frequencies) or given the graph but with bars not filled in. We need to work out the frequencies to fill in in the table and the frequency density to fill in the graph. For example

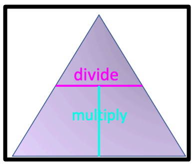


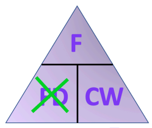
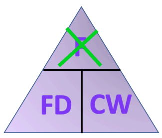
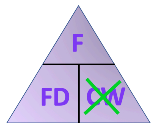
Time	Frequency
0 ≤ x < 20	15
20 ≤ x <40	
40 ≤ x <50	
50 ≤ x <56	
56 ≤ x < 60	
60 ≤ x < 70	45
70 ≤ x < 100	30

The bar from 0-20 is not drawn (we can use the table to work out the frequency density which is $\frac{\text{frequency}}{\text{width}}$) and the table is not filled in for times between 20 and 60 (we can use the graph to work out the frequencies. The frequencies are just the areas of the rectangles since re-arranging the formula $fd = \frac{\text{frequency}}{\text{class width}}$ gives frequency $=$ fd $\times$ class width).

We can use the following pyramids as a "hack" to help us remember the formulae. We cross off what we're trying to find and do the operation that the remaining 2 elements in the pyramid shows - multiplication if vertical line or division if horizontal line

key



To Find the Frequency Density	To Find the Frequency	To Find the Class Width
Recall that this tell us the height of the bars on the graph and we already know that this is $\frac{frequency}{class\ width}$ Using the pyramid confirms this  $FD = \frac{F}{CW}$	This tells us how many and also helps us fill in the table (frequency column) when we have gaps  $F = FD \times CW$ Note: We don't really need this formula as the frequency is just the area of the rectangles (bars) which is just length times width.	We don't really need this formula as we can just subtract the bar categories to get this, but it has been included here for completeness.  $CW = \frac{F}{FD}$

Be sure to close the gap if there is a gap between boundaries. We need to do this since the histograms are continuous so there shouldn't be a jump

Time	Closing The Gap	Class width
$0 < t \leq 6$	$-0.5 < t \leq 6.5$	7
$7 < t \leq 8$	$6.5 < t \leq 8.5$	2
$9 < t \leq 15$	$8.5 < t \leq 15.5$	7

Notice how there is no gap between the bars/rectangles once we close the gap

Let's look at examples of all types of questions

- Given the entire table draw graph
- Given part of table and graph, fill in the missing parts in the graph and table
- Given all of the graph but no table (only given the numbers of some of the bars)
- Finding the mean and median
- Proportional questions

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Given a table draw the graph

Example 1:

The table gives information about the speeds, in km/h of 81 cars measured on a motorway. On the grid draw the histogram for the information in the table

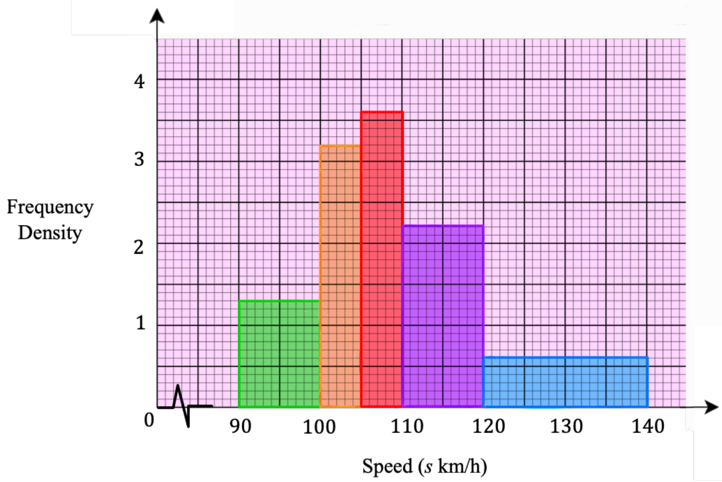
Speed (s km/h)	Frequency
$90 < s \le 100$	13
$100 < s \le 105$	16
$105 < s \le 110$	18
$110 < s \le 120$	22
$120 < s \le 140$	12

Answer

Let's find the frequency density for each row

Recall the formula: Frequency density $= \frac{frequency}{class\ width}$

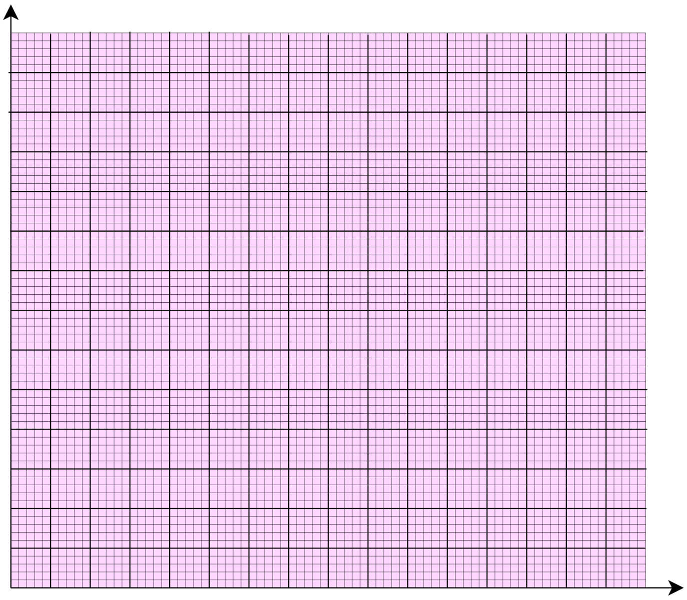
Speed (s km/h)	Frequency	Width	Frequency Density
$90 < s \le 100$	13	$100 - 90 = 10$	$13 \div 10 = 1.3$
$100 < s \le 105$	16	$105 - 100 = 5$	$16 \div 5 = 3.2$
$105 < s \le 110$	18	$110 - 105 = 5$	$18 \div 5 = 3.6$
$110 < s \le 120$	22	$120 - 110 = 10$	$22 \div 10 = 2.2$
$120 < s \le 140$	12	$140 - 120 = 20$	$12 \div 20 = 0.6$



Example 2:

The table gives information about waiting times in minutes at 25 railway crossings. On the grid draw the histogram for the information in the table.

Waiting Times	Frequency
2-4	5
5-6	4
7	1
8	6
9-10	9



Answer

Let's find the frequency density for each row using the formula $\frac{frequency}{class\ width}$

Waiting Times	Frequency	Width	Frequency Density
$1.5 - 4.5$	5	$4.5 - 1.5 = 3$	$5 \div 3 = 1.7$
$4.5 - 6.5$	4	$6.5 - 4.5 = 2$	$4 \div 2 = 2$
$6.5 - 7.5$	1	$7.5 - 6.5 = 1$	$1 \div 1 = 1.4$
$7.5 - 8.5$	6	$8.5 - 7.5 = 1$	$6 \div 1 = 6$
$8.5 - 10.5$	9	$10.5 - 8.5 = 2$	$9 \div 2 = 4.5$

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Given part of a table and graph, fill in the missing graph and table

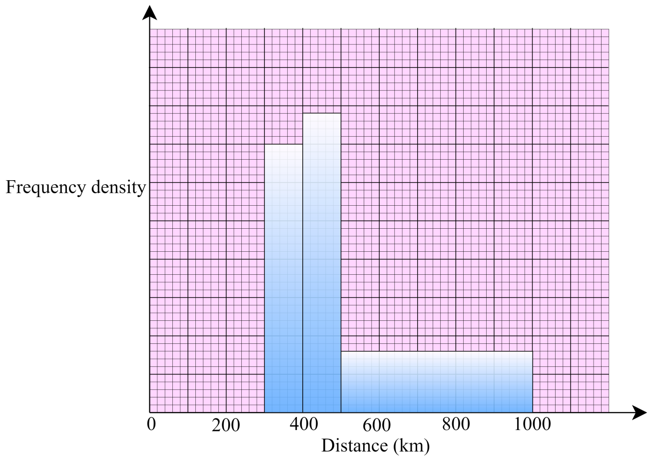
Example 1:

The incomplete table and histogram give some information about the distances walked by some students in a school in one year.

Distance (d) in km	Frequency
$0 < d \leq 300$	210
$300 < d \leq 400$	350
$400 < d \leq 500$	
$500 < d \leq 1000$	

i. Use the information in the histogram to complete the frequency table

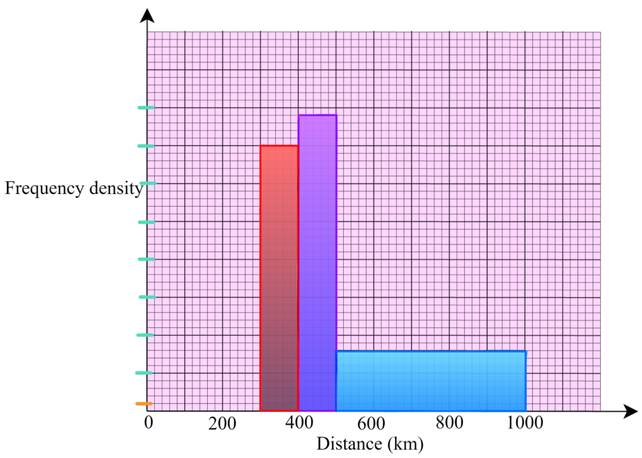
ii. Fill in the rest of the histogram



Answer

i.

Distance (d) in km	Frequency
$0 < d \leq 300$	210
$300 < d \leq 400$	350
$400 < d \leq 500$	
$500 < d \leq 1000$	



We use the red row to get the scale since this part of the histogram is already drawn.

$$FD = \frac{\text{Frequency}}{\text{Width}} = \frac{350}{100} = 3.5$$

Find the scale (y-axis)

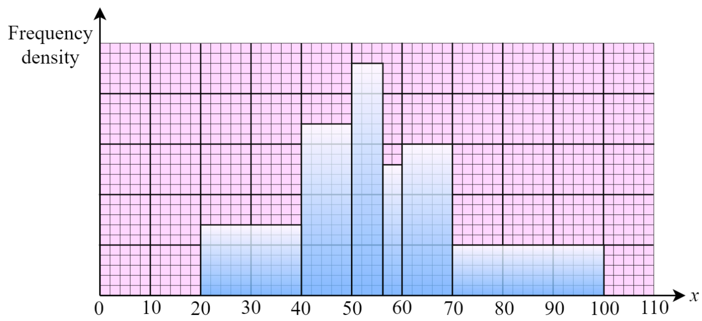
To get each scale (major tick): $\frac{3.5}{7} = 0.5$

To get each scale (minor tick): $\frac{0.5}{5} = 0.1$

Example 2:

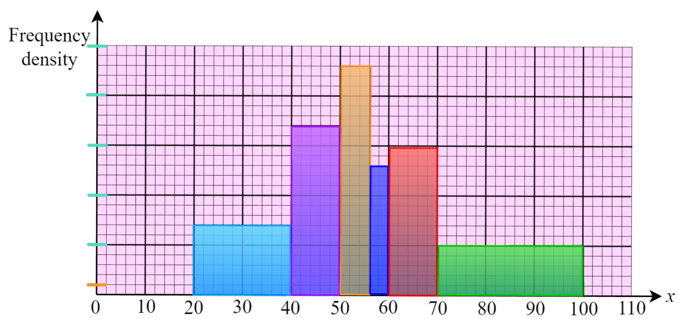
Use the information in the histogram to complete the frequency table and fill in the rest of the histogram

Time	Frequency
$0 \leq x < 20$	15
$20 \leq x < 40$	
$40 \leq x < 50$	
$50 \leq x < 56$	
$56 \leq x < 60$	
$60 \leq x < 70$	45
$70 \leq x < 100$	30



Answer

Time	Frequency
$0 \leq x < 20$	15
$20 \leq x < 40$	
$40 \leq x < 50$	
$50 \leq x < 56$	
$56 \leq x < 60$	
$60 \leq x < 70$	45
$70 \leq x < 100$	
$100 \leq x < 110$	30



We can use the red row to get the scale since this part of the histogram is already drawn (or we could have used the green row)

$$FD = \frac{\text{Frequency}}{\text{Width}} = \frac{45}{10} = 4.5$$

Find the scale (y-axis)

To get each scale (major tick): $\frac{4.5}{3} = 1.5$

To get each scale (minor tick): $\frac{1.5}{5} = 0.3$

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To fill in the rest of the table we need the frequencies which are the area of each of the rectangles

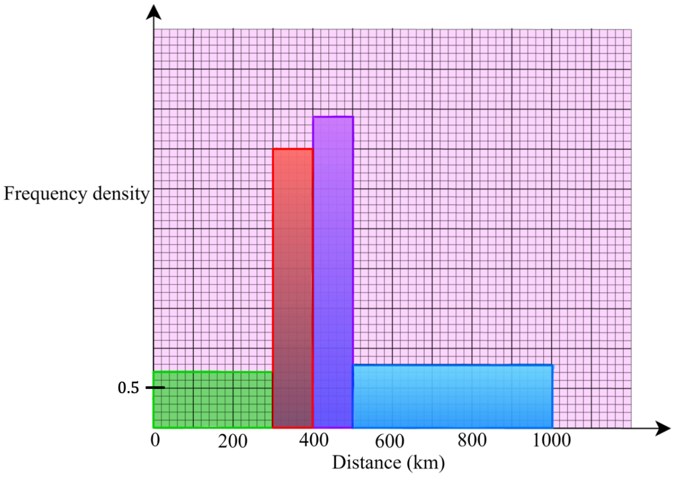
$$F = 100 \times 3.9 = 390$$
$$F = 500 \times 0.8 = 400$$

So, we have

Distance (d) in km	Frequency
0 < d ≤ 300	210
300 < d ≤ 400	350
400 < d ≤ 500	390
500 < d ≤ 1000	400

ii.

Distance (d) in km	Frequency	Frequency Density
0 < d ≤ 300	210	$210 \div 300 = 0.7$
300 < d ≤ 400	350	$350 \div 100 = 3.5$
400 < d ≤ 500	390	$390 \div 100 = 3.9$
500 < d ≤ 1000	400	$400 \div 500 = 0.8$



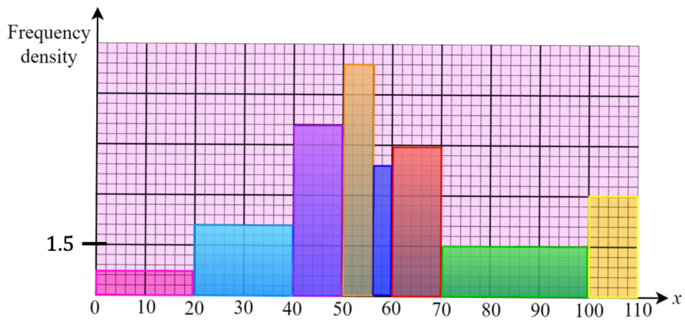
To fill in the rest of the table we need to frequencies which are the area of each of the rectangles

$$F = 20 \times 2.1 = 42$$
$$F = 10 \times 5.1 = 51$$
$$F = 6 \times 6.9 = 41.1$$
$$F = 4 \times 3.9 = 15.6$$
$$F = 30 \times 1.5 = 45$$

So, we have

Time	Frequency
0 ≤ x < 20	15
20 ≤ x <40	42
40 ≤ x <50	51
50 ≤ x <56	41.1
56 ≤ x < 60	15.6
60 ≤ x < 70	45
70 ≤ x < 100	45
100 ≤ x < 110	30

Time	Frequency	Frequency Density
0 ≤ x < 20	15	$15 \div 20 = 0.75$
20 ≤ x <40	42	$42 \div 20 = 2.1$
40 ≤ x <50	51	$51 \div 10 = 5.1$
50 ≤ x <56	41.1	$41.4 \div 5 = 6.93$
56 ≤ x < 60	15.6	$15.6 \div 4 = 3.9$
60 ≤ x < 70	45	$45 \div 10 = 4.5$
70 ≤ x < 100	45	$45 \div 30 = 1.5$
100 ≤ x < 110	30	$30 \div 10 = 3$



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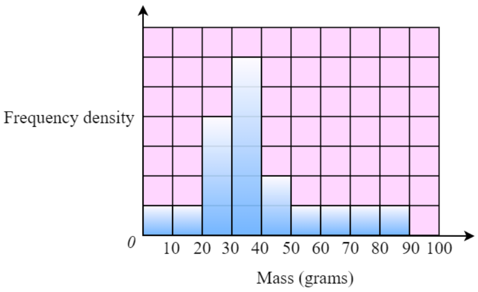
Given graph but no table and no scale

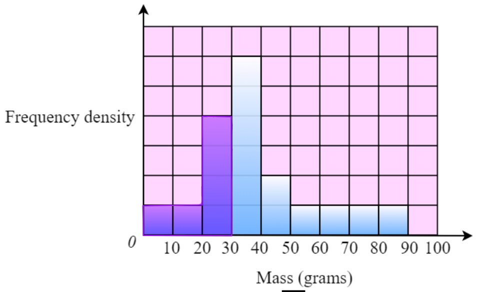
Example 1:

The histogram shows information about the masses, in grams, of some stones. There are 120 stones with masses less than 30 g. Calculate an estimate for the number of stones with masses between 35 g and 70 g.



Answer

Consider stones of masses less than 30g:



Let's count how squares of size that we have in this region

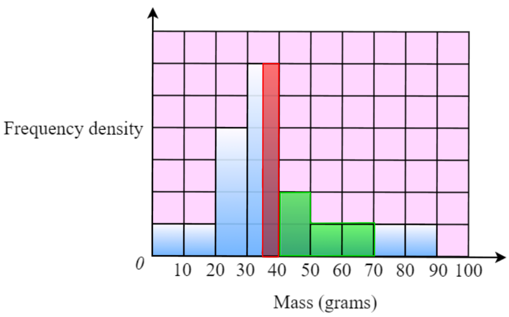
There are 6 squares

We are told that these 6 squares represents 120

Hence one purple square represents,

$120 \div 6 = 20$ stones per square

Let's look at masses between 35g and 70g :



There are:

6 half squares: $0.5 \times 6 = 3$ squares

4 squares

In total:

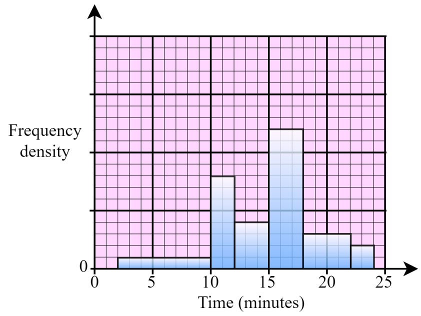
$3 + 4 = 7$ squares

Each square represents 20 stones

$7 \times 20 = 140$ stones

Example 2:

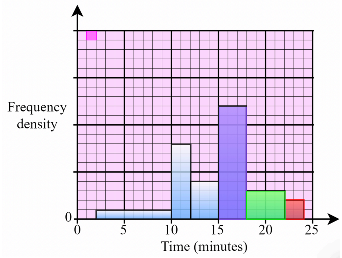
The histogram shows the times taken to complete a crossword by a random sample of students. The number of students who completed the crossword in more than 15 minutes is 78. Estimate the percentage of students who took less than 11 minutes to complete the crossword.



Answer

Way 1:

Consider the number of students who completed the crossword in more than 15 minutes



Let's count how many squares of size in this region

$3 \times 12 = 36$ tiny squares

$4 \times 3 = 12$ tiny squares

$2 \times 2 = 4$ tiny squares

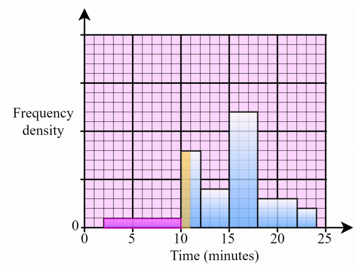
$36 + 12 + 4 = 52$ tiny squares

We are told that these 52 squares represent 78

Hence one square represents,

$78 \div 52 = 1.5$ people per square

Let's now consider those who took less than 11 minutes

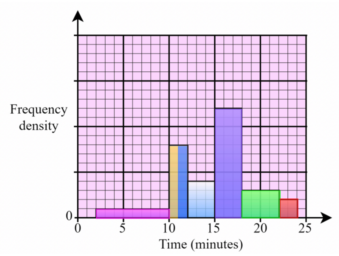


$8 \times 1.5 = 12$

$8 \times 1.5 = 12$

Total number of students who took less than 11 minutes

$12 + 12 = 24$



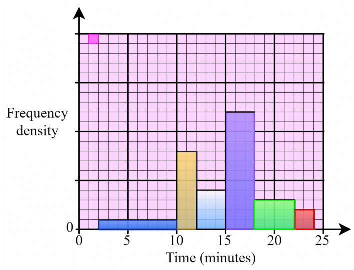
Now we need the total number of students

$12+12+12+(12 \times 1.5)+(36 \times 1.5)+(12 \times 1.5)+(4 \times 1.5) = 132$

$\text{percentage} = \frac{24}{132} \times 100 = 18.18\%$

Way 2:

Since the question wants a percentage, we didn't actual need to be given the total numbers in this question. We could have just counted areas



$2 - 10$: 8 squares

$10 - 12$: 16 squares

$12 - 15$: 12 squares

$15 - 18$: 36 squares

$18 - 22$: 12 squares

$22 - 24$: 4 squares

The percentage who took less than 11 minutes

$$\frac{8 + \frac{1}{2}(16)}{8 + 16 + 12 + 36 + 12 + 4} \times 100$$

$$= \frac{16}{88} \times 100$$

$$= 18.18\%$$

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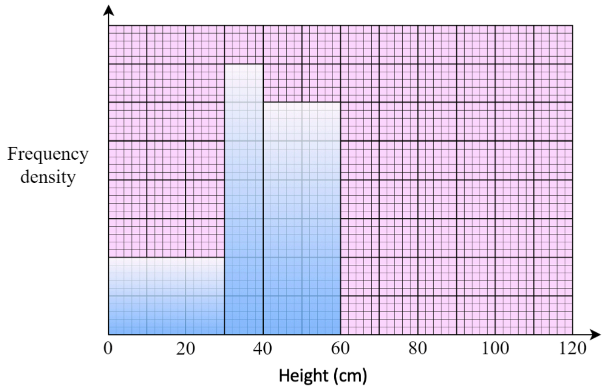
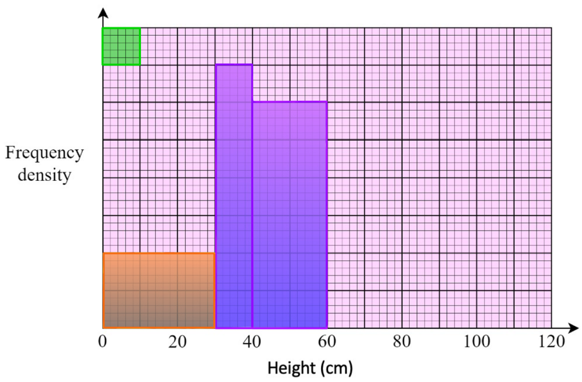
Example 3:

The histogram gives information about the heights of some plants.
There are 360 plants with a height of 30 cm or less

i. Work out the number of plants with a height of more than 30 cm
ii. What percent have a height between 40 and 50 cm



Answer



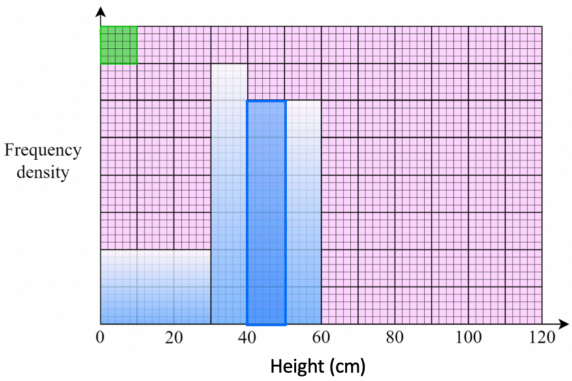
The orange region represents 360 plants

The means that a green square represents $\frac{360}{6} = 60$ plants

More than 30 cm is represented by the purple region which has 19 green squares:

$$19(60) = 1140 \text{ plants}$$

ii.

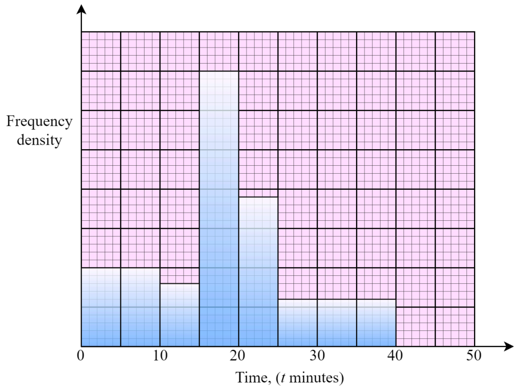


Between 40 and 50 is the dark blue region which is 6 green squares
In total there are 25 green squares

$$\frac{6}{25} \times 100 = 24\%$$

Example 4:

The histogram shows information about the time, t minutes, patients spent at a doctors' surgery on one day. No patient spends more than 40 minutes at the surgery.

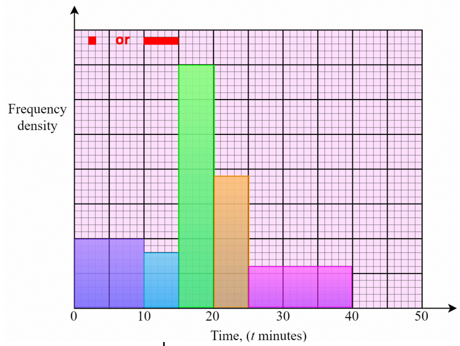


i. Calculate the percentage of the patients who spent between 25 and 40 minutes at the surgery

16 patients spent between 10 and 15 minutes at the surgery.

ii. Calculate the total number of patients at the surgery that day

Answer



Let's count the number of small red squares in each region

$10(10) = 100$ squares
$5(8) = 40$ squares
$5(35) = 175$ squares
2 $5(19) = 95$ squares
$15(6) = 90$ squares

$$\frac{90}{100 + 40 + 175 + 95 + 90}$$

$$= \frac{90}{500} = 18\%$$

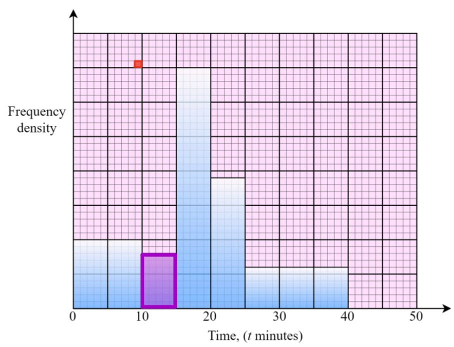
Let's count the number red **sticks** in each region

four 5x5 squares = 20 sticks
one 5x5 squares and 3 sticks =8 sticks
seven 5x5 squares = 35 sticks
three 5x5 squares and 4 sticks = 19 sticks
twenty 23 5x5 squares and 3 sticks =18 sticks

$$\frac{18}{20 + 8 + 35 + 19 + 18} = \frac{18}{100} = 18\%$$

ii.

The area shown in purple represents 16 people.



There are 40 little squares in the purple region

One tiny red square represents $\frac{16}{40} = 0.4$

There are 500 red squares in total

$500(0.4) = 200$ people

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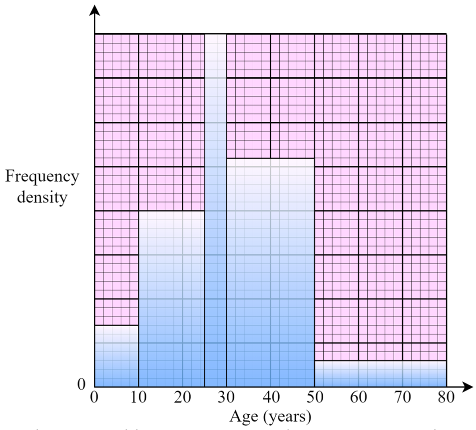
Given graph but no table (given scale/can work out scale)

Example 1:

The histogram shows information about the ages of the members of a football supporters club.

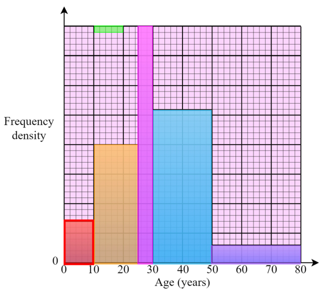


There are 20 members aged between 25 and 30. One member of the club is chosen at random. What is the probability that this member is more than 30 years old?

Answer

Way 1:

Work with the areas – we don't even need to know that 20 members were between 25 and 30



Let's count the number green sticks

$0-10$: 7 sticks

$10-20$: 6 5x5 squares which is 30 sticks

$25-30$: 4 5x5 squares which is 20 sticks

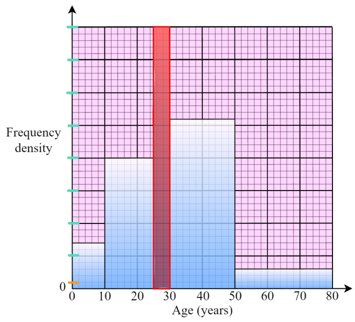
$30-50$: 10 5x5 squares and 2 sticks $=$ 52 sticks

$50-80$: 9 sticks

$$\frac{52+9}{7+30+20+52+9}=\frac{61}{118}$$

Way 2:

You can also find the scale instead



We are told that the red region represents 20 people i.e. has an frequency of 20

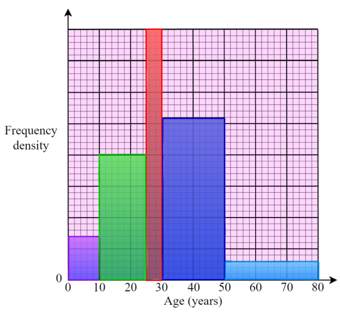
So, let's use the red bar to help us get the scale:

$$FD=\frac{Frequency}{width} \Rightarrow FD=\frac{20}{5}=4$$

$$4 \div 8 = 0.5$$

$$0.5 \div 5 = 0.1$$

Now we find the frequency for each group (area of each rectangle)



$$10 \times 0.7 = 7$$

$$15 \times 2 = 30$$

$$20 \times 2.6 = 52$$

$$30 \times 0.3 = 9$$

Now we can find the probability:

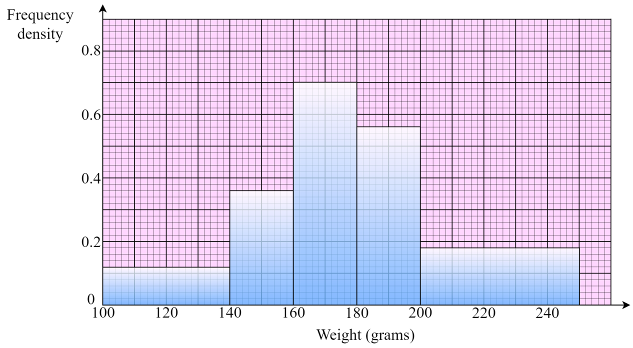
P(more than 30 years old)

$$=\frac{\text{Total freq more than 30}}{\text{Total freq}}$$

$$=\frac{(52+9)}{(7+30+20+52+9)}=\frac{61}{118}$$

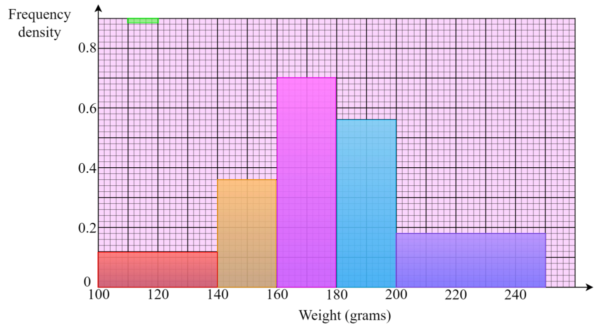
Example 2:

The histogram shows some information about the weights of a sample of apples. Work out the proportion of apples in the sample with a weight between 140 grams and 200 grams



Answer

Way 1: Count the areas



Let's count the number green sticks

14 5x5 squares and 4 sticks which is 24 sticks

6 5x5 squares and 6 sticks which is 36 sticks

14 5x5 squares which is 70 sticks

10 5x5 squares and 6 sticks which is 56 sticks

5 5x5 squares and 20 sticks which is 45 sticks

$$\frac{36+70+56}{24+36+70+56+45}=\frac{162}{231}=70\%$$

Way 2:

We have the scale so we can build a table

Weight (grams)	Frequency
$100 < x \leq 140$	$0.12 \times 40 = 4.8$
$140 < x \leq 160$	$0.36 \times 20 = 7.2$
$160 < x \leq 180$	$0.7 \times 20 = 14$
$180 < x \leq 200$	$0.56 \times 20 = 11.2$
$200 < x \leq 250$	$0.18 \times 50 = 9$

Total $= 46.2$

$140 < x \leq 200 = 7.2 + 14 + 11.2 = 32.4$

Therefore,

$$\frac{32.4}{46.2} \times 100 = 70\%$$

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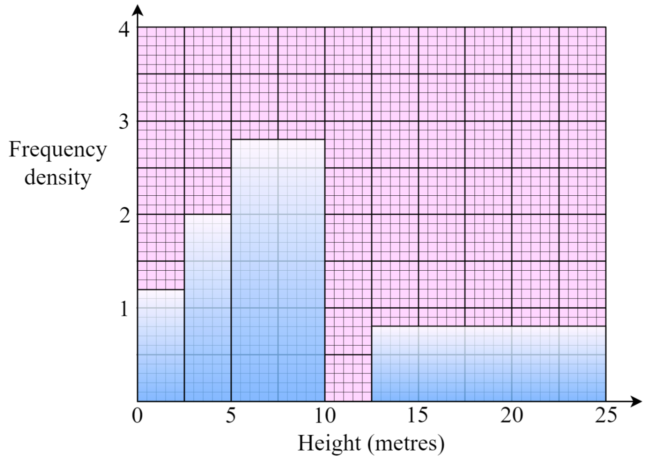
Example 3:

The histogram gives information about the heights, in metres, of the trees in a park. The histogram is incomplete.



20% of the trees in the park are between 10 metres and 12.5 metres.
None of the trees in the park have a height greater than 25 metres.
Complete the histogram.

Answer

We have the scale so we don't need to count areas. We can build a table

Height (metres)	Frequency
$0 < x \leq 2.5$	$1.2 \times 2.5 = 3$
$2.5 < x \leq 5$	$2 \times 2.5 = 5$
$5 < x \leq 10$	$2.8 \times 5 = 14$
$10 < x \leq 12.5$	
$12.5 < x \leq 25$	$0.8 \times 12.5 = 10$

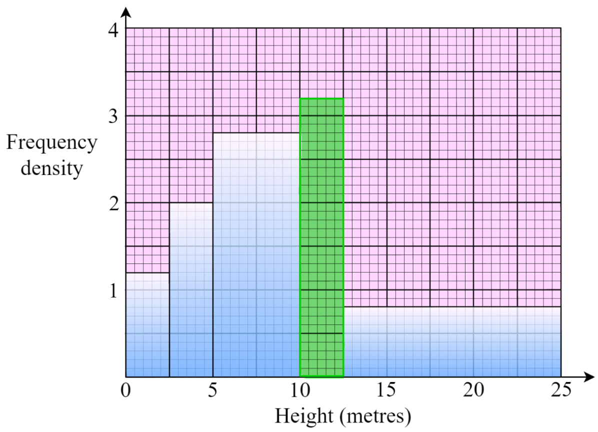
If $10 < x \leq 12.5$ represents 20%, then the rest represent 80%
The total number of the rest is 3+4+14+10=32
So, 32 trees represents 80%

$$80\% = 32$$
$$20\% = 8$$

$FD = \frac{frequency}{width}$

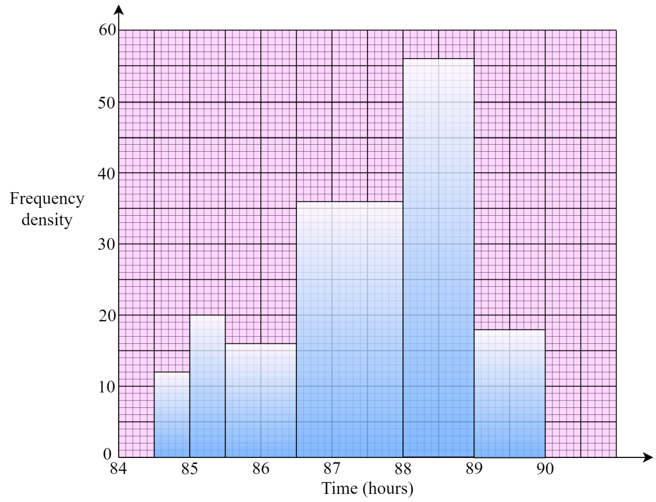
Hence,

$FD = \frac{8}{2.5} = 3.2$



Example 4:

The histogram shows information about the times taken by 160 cyclists to complete the Tour de France cycle race. 6 cyclists took less than 85 hours.

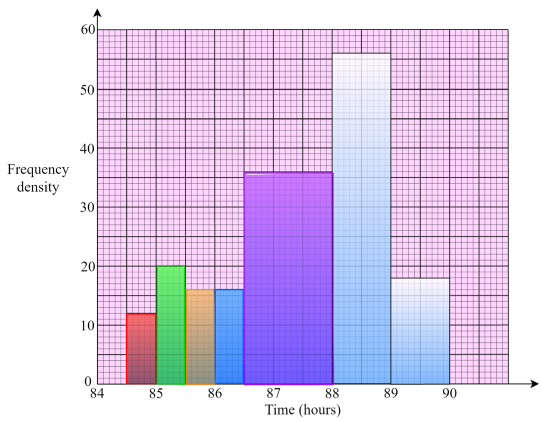


i. Work out an estimate for the number of cyclists who took less than 86 hours
ii. For these 160 cyclists, work out an estimate for the time taken by the cyclist who finished in 50th position

Answer

We are given the scale for this question, so don't need to count areas

i.



$$\text{Frequency} = \text{FD} \times \text{Width}$$

$12 \times 0.5 = 6$ (this is in keeping with what we are told in the question, so this means we don't need to scale up or down. The area is the exact frequency)

$$20 \times 0.5 = 10$$
$$16 \times 0.5 = 8$$
$6 + 10 + 8 = 24$ people

ii.

Where does the 50th person occur? Let's count up to see where they occur.

$$12 \times 0.5 = 6$$
$$20 \times 0.5 = 10$$
$$16 \times 0.5 = 8$$
$$16 \times 0.5 = 8$$

$$6 + 10 + 8 + 8 = 32$$

We need 18 more to get to 50

Let's consider what the purple represents: $1.5 \times 36 = 54$

$\frac{1}{3}$ of this is exactly 18 which is at 87

87 hours

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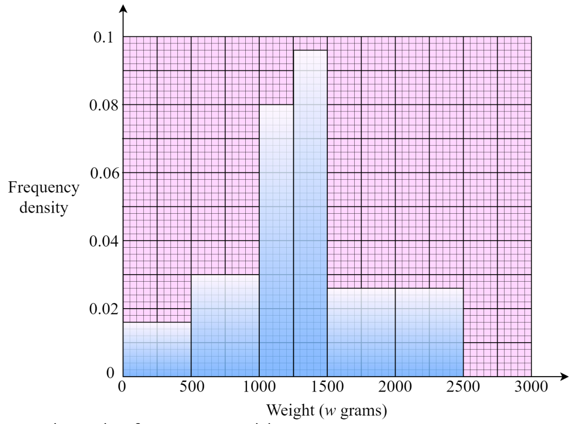
Mean and Median

Example 1:

Jim went on a fishing holiday. The histogram shows some information about the weights of the fish he caught



Weight (w grams)	Frequency
0 <w ≤ 500	8
500 < w ≤ 1000	
1000 < w ≤ 1250	
1250 < w ≤ 1500	
1500 < w ≤ 2500	

i. Use the histogram to complete the frequency table

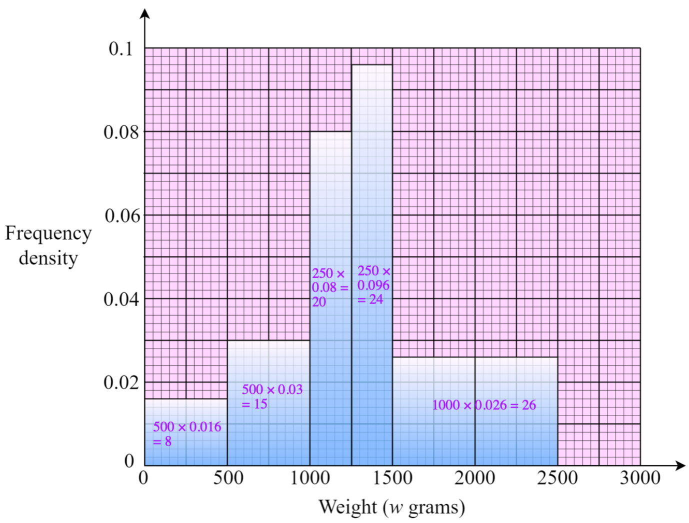
Jim kept all the fish he caught with a weight greater than 2000 g.

ii. Find the ratio of the number of fish Jim kept to the total number of fish he caught

iii. Use the histogram to find an estimate of the median

i.

we have the scale so we can easily work out the frequencies and hence fill the table in

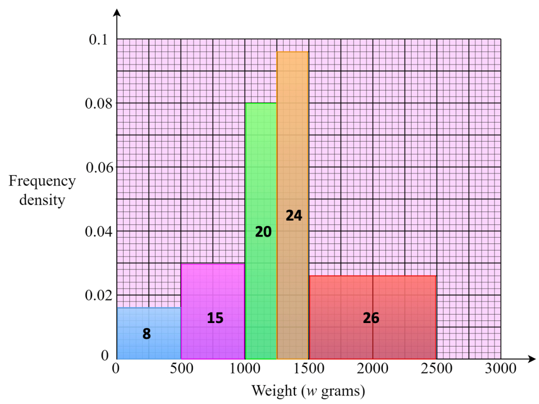


Weight (w grams)	**Frequency**
0 <w ≤ 500	**8**
500 < w ≤ 1000	**15**
1000 < w ≤ 1250	**20**
1250 < w ≤ 1500	**24**
1500 < w ≤ 2500	**26**

ii.

$$0.026 \times 500 = 13$$
$$8 + 15 + 20 + 24 + 26 = 93$$
$$13 : 93$$

iii. we need to count up until we get to the middle value



total$= 8 + 15 + 20 + 24 + 26 = 93$

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Way 1: GCSE use $\frac{n+1}{2}$

Using $\frac{94}{2} = 47^{th}$ value

We can count from the top or the bottom to get as close to 47 as possible

Counting from the bottom, there are 43 in the first 3 bars, so the median must occur in the 4th bar. We need the weight of the 4th fish (47—43) in the 4th bar

Since the frequency of that group is 24, and the width is 250, the median is:

$$4 \times \frac{250}{24} = \frac{125}{3}\ g$$

Hence,

$$1250 + \frac{125}{3} = 1292\ g$$

Way 2: A Level use $\frac{n}{2}$

Using $\frac{93}{2} = 46.5^{th}$ value

We can count from the top or the bottom to get as close to 46.5 as possible

Counting from the bottom, there are 43 in the first 3 bars, so the median must occur in the 4th bar. We need the weight of the 3.5th fish (46.5—43) in the 4th bar

Since the frequency of that group is 24, and the width is 250, the median is:

$$3.5 \times \frac{250}{24} = \frac{875}{24}\ g$$

Hence,

$$1250 + \frac{875}{24} = 1286\ g$$

Way 3: Memorised Formula (Best Way)

$$LCB + \frac{\textbf{how many in}}{\textbf{group total}} \times \textbf{class width}$$

Note: LCB means lower class boundary

$$\frac{n}{2} = \frac{93}{2} = 46.5^{th} \text{ value}$$

We need the 46.5 th value

This is in the 4th bar

Weight	f	UCB	cf
0 - 500	8	500	8
500 -1000	15	1000	23
1000 - 1250	20	1250	43
1250 - 1500	24	1500	67
1500 -2500	26	2500	93

$$1250 + \frac{46.5 - 43}{24} \times 250 = 1286.5$$

Way 4: Interpolation Definition Formula

Weight	f	UCB	cf
0 - 500	8	500	8
500 -1000	15	1000	23
1000 - 1250	20	1250	43
1250 - 1500	24	1500	67
1500 -2500	26	2500	93

$\frac{93}{2} = 46.5^{th}$ value .

Zoom in on the yellow

1250	43
x	46.5
1500	67

$$\frac{x-1250}{1500-1250} = \frac{46.5-43}{67-43}$$

$$\frac{x-1250}{250} = \frac{3.5}{24}$$

$$x - 1250 = 3.646$$

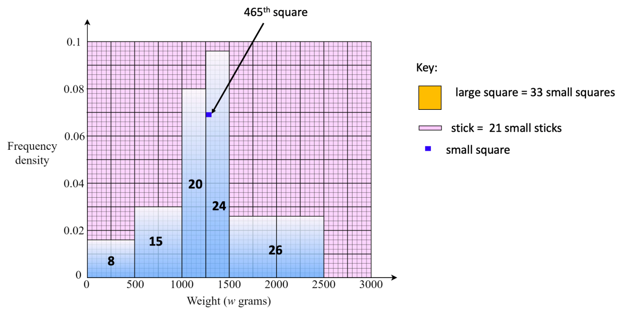
$$x = 1286\ g$$

Way 5: Count the tiny squares until you reach the middle (this is not very accurate)

There are 33(25)+21(5)= 930 little squares in total

$$\frac{930}{2} = 465$$

Count up form the bottom (or down from the top) until you get to 465 little squares and see which speed this gives you.



Roughly 1300 but this isn't accurate

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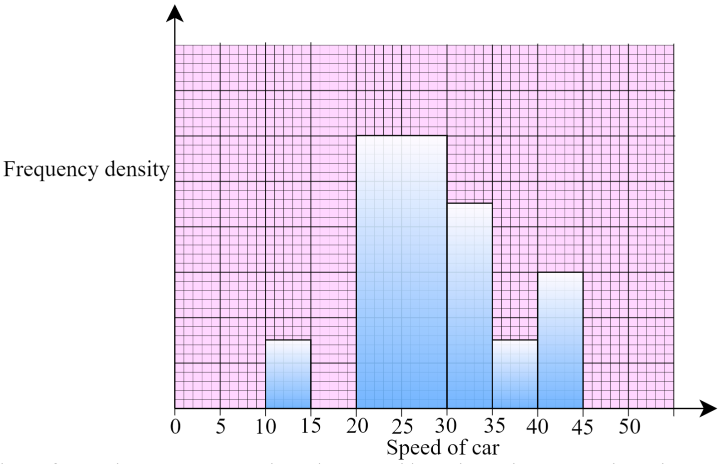
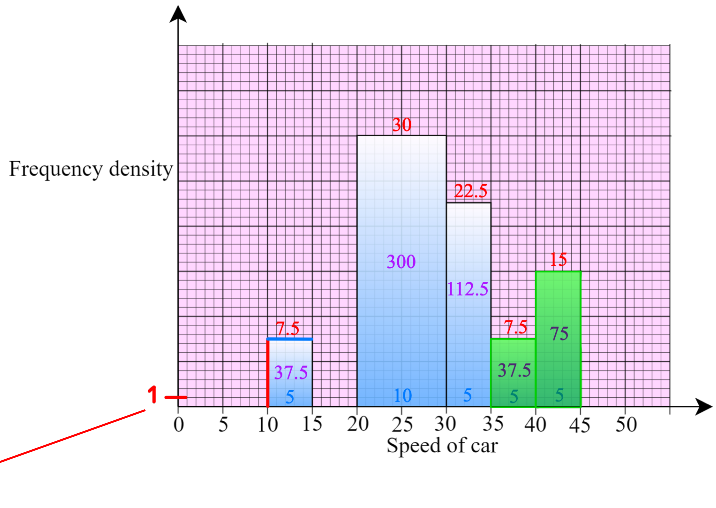
Example 2:
A policeman records the speed of the traffic on a busy road with a 30 mph speed limit. He records the speeds of a sample of 450 cars. The histogram below represents the results.



i. Calculate the number of cars that were exceeding the speed limit by at least 5 mph in the sample
ii. Estimate the value of the mean speed of the cars in the sample
iii. Estimate, to 1 decimal place, the value of the median speed of the cars in the sample

We don't know the scale and only know the total number. This is fine though. We can call the scale anything.



Just call this 1 for now. We will re-scale after if necessary.
It doesn't matter that we pick anything for the scale, choose any scale at first and then we have to check after what the scale up or down is.

Find the area of the shapes to get the purple numbers in each rectangle
Total area $= 37.5 + 300 + 112.5 + 37.5 + 75 = 562.5$
This area represents 450 cars
$sf = \frac{562.5}{450} = 1.25$ (this is what we re-scale by)

Exceeding the speed limit by at least 5 mph means greater than 35 mph
Area of the shaded bars $= 112.5$

$$\frac{112.5}{1.25} = 90 \text{ cars}$$

Note: There was also another way to get the scale
Total area $= 450$ (given in question)
We need to count how many green sticks

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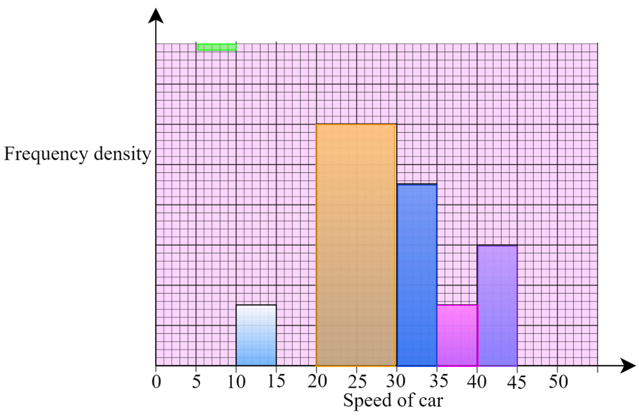

Total number of sticks: 7.5 + 60 + 22.5 + 7.5 + 15 = 112.5

We know these 112.5 sticks represents 450 cars in total

$\frac{450}{112.5} = 4$

So 1 stick represents 4 cars

This means that the area of the rectangle from 10-15 is 30 since it has 7.5 sticks and each stick represents 4 cars hence 7.5(4)=30

The width is of the rectangle is 5, let's call the height x

$5x = 30$

$x = 6$

So, the height of the rectangle from 10-15 is 6 which means each little scale tick is $\frac{16}{7.5} = 0.8$, not 1

i.

Speed	Midpoint	f
$10 - 15$	12.5	$\frac{37.5}{1.25} = 30$
$20 - 30$	25	$\frac{300}{1.25} = 240$
$30 - 35$	32.5	$\frac{112.5}{1.25} = 90$
$35 - 40$	37.5	$\frac{37.5}{1.25} = 30$
$40 - 45$	42.5	$\frac{75}{1.25} = 60$

$$\frac{(30 \times 12.5) + (240 \times 25) + (90 \times 32.5) + (30 \times 37.5) + (60 \times 42.5)}{450} = \frac{12974}{450} = 28.8$$

ii. Let's build a table to find the median and interpolate.

we need to find the UCB and cf columns first

Also there is a gap in the histogram. We still need to account for the 15-20 so let's add that row with (there will be no change in frequency)

Speed	UCB	cf
$10 - 15$	15	30
$15 - 20$	20	30
$20 - 30$	30	270
$30 - 35$	35	360
$35 - 40$	40	390
$40 - 45$	45	450

Median occurs in here: $\frac{450}{2} = 225$

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Zoom in on the orange

20	**30**
x	**225**
30	**270**

$$\frac{x-20}{10}=\frac{195}{240}$$

$$x=28.1$$

Note: Not Re-scaling wouldn't have made a difference to the mean and median. We only have to re-scale when finding total numbers. Let's see this with not re-scaling and see how we get the same answer.

Speed	f	Midpoint
$10-15$	37.5	12.5
$20-30$	300	25
$30-35$	112.5	32.5
$35-40$	37.5	37.5
$40-45$	75	42.5

Speed	f	UCB	cf
$10-15$	37.5	15	37.5
$15-20$	0	20	37.5
$20-30$	300	30	337.5
$30-35$	112.5	35	450
$35-40$	37.5	40	487.5
$40-45$	75	45	562.5

$$\text{Mean}=\frac{(37.5\times 12.5)+(300\times 25)+(112.5\times 32.5)+(37.5\times 37.5)+(75\times 42.5)}{562.5}=\frac{16218.75}{562.5}=28.8$$

Median $\frac{562.5}{2}=281.25th$ value

Zoom in on the orange

20	**37.5**
x	**281.25**
30	**337.5**

$$\frac{x-20}{10}=\frac{243.75}{300}$$

$$x=28.1$$

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Proportional

The histogram shows the variable t which represents the time taken, in seconds, by a group of children to solve a puzzle. The shaded bar A represents a frequency of 78 children.

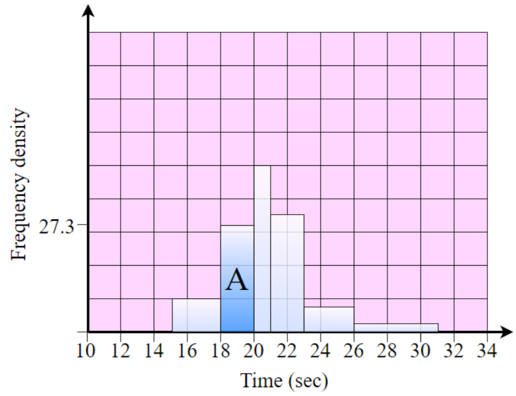
i. Why should a histogram be used to represent this data?

ii. Write down the underlying feature associated with each of the bars in a histogram

iii. What area on the histogram represents one child?

The total area under the histogram is 210 $units^2$

iv. Find the total number of children in the group



Answer

i. Time is continuous

ii. The area of the bar is proportional the frequency

iii. Area of bar A$= 2(27.3) = 54.6$ units^2

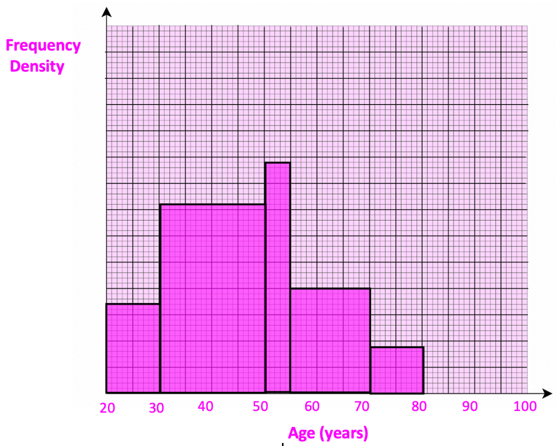
54.6 units^2 represents 78 children

We want the area for 1 child

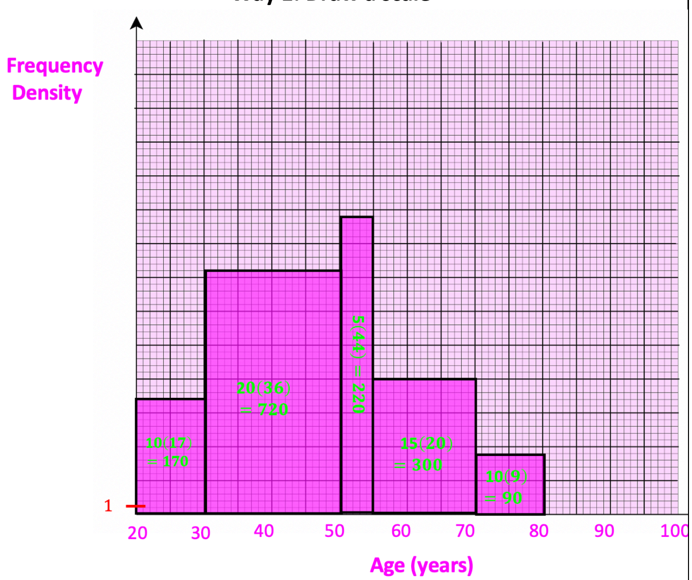
1 child $\frac{54.6}{78} = 0.7$ units^2

$\frac{210}{0.7} = 300$ children

150 people took part in a survey. The histogram shows information about the ages of these people. Work out how many of these 150 people are aged between 50 years and 55 years.



Way 1: Draw a scale

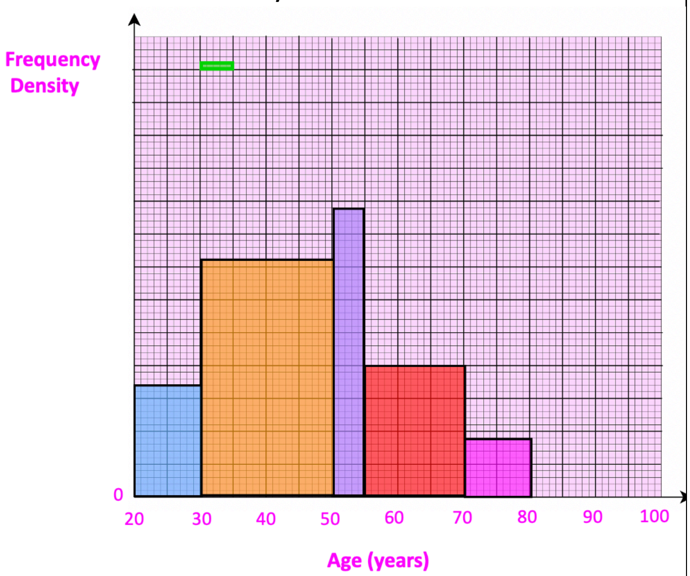


Just call the scale 1 as shown (we will re-scale after if necessary)

Find the area of the rectangles

Total area $= 1500$

Way 2: Count Areas



Let's count the number green sticks

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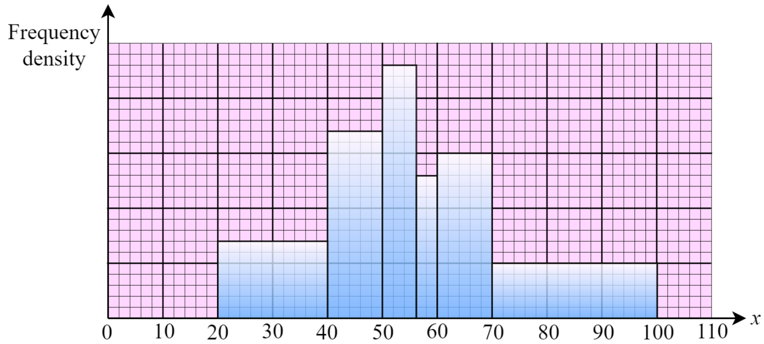
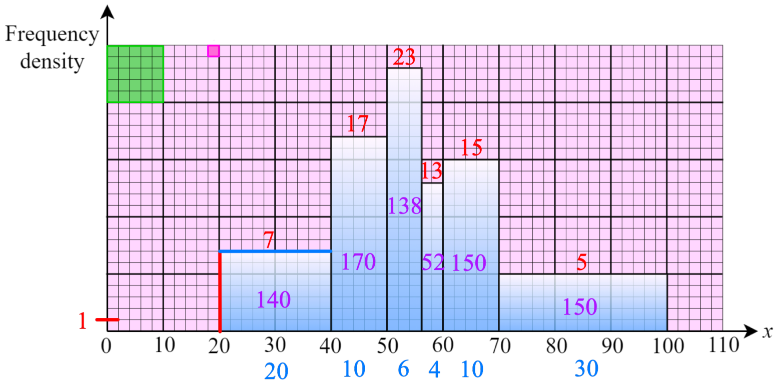
We know the real total area should have been 150 $sf = \frac{1500}{150} = 10$ (this is what we re-scale by) Between 50 and 56 Area of the bar between 50 and 55 $= 220$ $\frac{220}{10} = 22$ people	$20-30$: This is the equivalent of 6 5x5 squares and 4 sticks which is 34 sticks $30-50$: This is the equivalent of 28 5x5 squares and 4 sticks which is 144 sticks $50-55$: This is the equivalent of 8 5x5 squares and 4 sticks which is 44 sticks $55-70$: This is the equivalent of 12 5x5 squares which is 60 sticks $70-80$: This is the equivalent of 2 5x5 squares and 8 sticks which is 18 sticks In total there are 300 sticks representing 150 people $\frac{150}{300} = 0.5$ So each stick represents 0.5 people We want the 50-55 range which is 44 sticks $44(0.5) = 22$ people

The masses, x grams, of 800 apples are summarised in the histogram



i. Find the scale for the frequency density
ii. Find an estimate for the median number of apples



Just call this 1 for now. We will re-scale after if necessary.
It doesn't matter that we pick anything for the scale, choose any scale at first and then we have to check after what the scale up or down is.

Find the area of the shapes to get the purple numbers in each rectangle

Total: $140 + 170 + 138 + 52 + 150 + 150 = 800$

$\frac{800}{800} = 1$, so don't need to rescale

Given in question.

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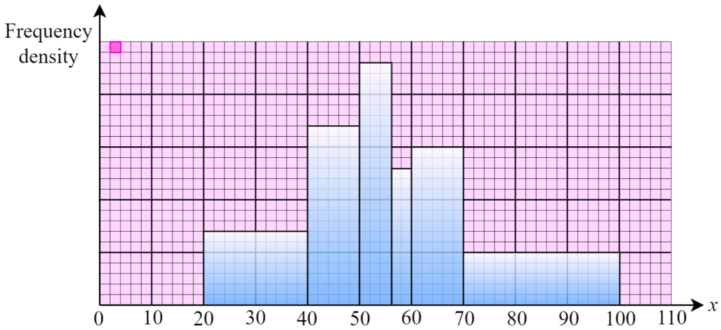
Let’s build a table to find the median

Weight (w grams)	Frequency
20 <x ≤ 40	140
40 <x ≤ 50	170
50 <x ≤ 56	138
56 <x ≤ 60	52
60 <x ≤ 70	150
70 <x ≤100	150

Note: There was also another way to get the scale
Total area $= 800$ (given in question)
We need to count how many ‘s



Total number of little pink squares: $20(7) + 10(7) + 5(22) + 5(12) + 10(15) + 30(5) = 400$

There are 400 little ’s

We know this represents 800 apples in total
$\frac{800}{400} = 2$

So 1 represents 2 apples

This means that the area of the rectangle from 20-40 is 140 since it has 10(7)=70 little blocks and each block represents 2 apples hence 2(70)=140
The width is of the rectangle is 20, let’s call the height x
$20x = 140$
$x = 7$
So, the height of the rectangle from 20-40 is 7 which means each little scale tick is 1

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Proportional bars

The table shows the distances, to the nearest km, travelled to work each of 50 employees in an office.
A histogram is drawn to represent these data. The bar representing the distance 3-5 has a width of 1.5 cm and a height of 6 cm. Calculate the width and height of the bar representing 6-10.

Distance (km)	Frequency
0-2	16
3-5	12
6-10	10
11-20	8
21-40	4

Answer

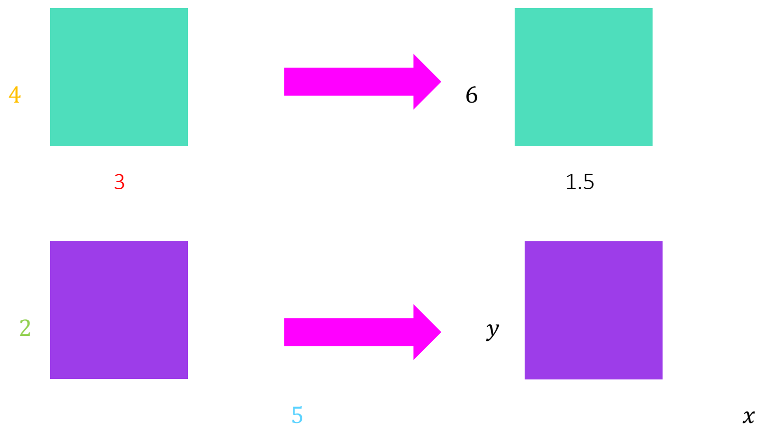
We need to close the gaps first between the boundaries (the upper class of any row and the lower class of the subsequent row). We use our knowledge of bounds to do this

Distance (km)	Distance (km) (closing the gap)
0-2	−0.5-2.5
3-5	2.5-5.5
6-10	5.5-10.5
11-20	10.5-20.5
21-40	20.5-40.5

Now let's find the frequency densities (FD) which is the frequency divided by the width.

Distance (km)	Frequency	Width	FD
0-2.5	16		
2.5-5.5	12	3	$\frac{12}{3}=4$
5.5-10.5	10	5	$\frac{10}{5}=2$
10.5-20.5	8		
20.5-40.5	4		

The question concentrates on the green and purple rows since it talks about the bar representing the width of 3-5 and the bar representing 6-10.
Let's draw the rectangles out from 2 rows in the table on the left-hand side below (dimensions are the width and frequency density) and then the scaled versions for each on the right-hand side below (dimensions are given in the question).



Use similar shapes to find x and then do the same to find y

$\frac{4}{6}=\frac{2}{y}$ $y=3\ cm$ (height)	$\frac{3}{1.5}=\frac{5}{x}$ $x=2.5\ cm$ (width)

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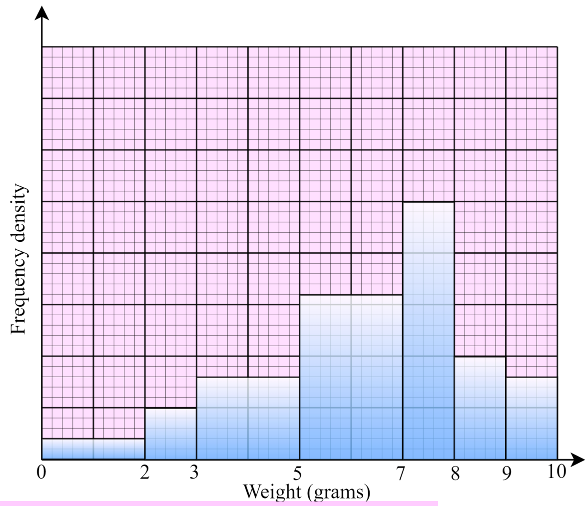
GCSE

7
8
9
10
11 find scale
12 fin scale
16 find scale
17

20 proportional
25 median
30 no scale
32 o scale
25 a level sraw bas

1) Ralph records the weights in grams, of 100 tomatoes. This information is displayed in the histogram below.



Given that 5 of the tomatoes have a weight between 2 and 3 grams

i. find the number of tomatoes with a weight between 0 and 2 grams.

One of the tomatoes is selected at random

ii. Find the probability that it weighs more than 3 grams

iii. Estimate the proportion of the tomatoes with a weight greater than 6.25 grams.

iv. Explain whether the median is less than or greater than 6.25 grams.

The mean weight of these tomatoes is 6.25 grams. Two of these 100 tomatoes are selected at random.

iv. Estimate the probability that both tomatoes weigh within 0.75 grams of the mean.