Definitions

Consider the following table of frequencies:

x Test Score (%)	Frequency
$0 < x \leq 10$	2
$10 < x \leq 30$	8
$30 < x \leq 60$	3
$60 < x \leq 70$	5

For example:
The 2 tells us that 2 people scored between 0% and 10%
The 8 tells us that 8 people scored between 10% and 30%
The 3 tells us that 3 people scored between 30% and 60%
The 5 tells us that 5 people scored between 60% and 70%

If given the table above of frequencies, we can find the cumulative frequency (cf). Cumulative frequency is just the running total of all the frequencies.

x Test Score (%)	Cumulative Frequency
$0 < x \leq 10$	2 (copy the first frequency)
$0 < x \leq 30$	$2 + 8 = 10$
$0 < x \leq 60$	$10 + 3 = 13$
$0 < x \leq 70$	$13 + 5 = 18$ (this is the total number)

For example:
The 2 tells us that 2 people scored below 10%
The 10 tells us that 10 people scored below 30%
The 13 tells us that 13 people scored below 60%
The 18 tells us that 18 people scored below 70%

Take note of:

- Always starting from zero in the table for the lower boundary (it is a running total, so we start from the beginning each time)
- The **upper-class boundaries** which are 10, 30, 60 and 70 (this will be useful for when we graph a cumulative frequency curve)

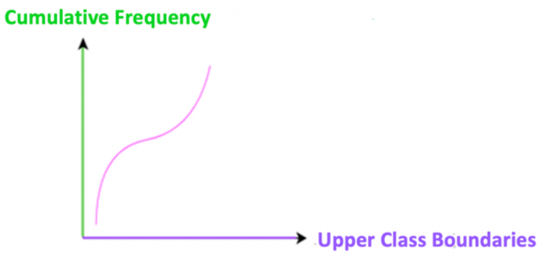
We can also find the relative cumulative frequencies which tell us the percentage of the total. Relative frequencies are very rarely used though.

x Test Score (%)	Relative Cumulative Frequency
$0 < x \leq 10$	$\frac{2}{18} = 0.11$
$0 < x \leq 30$	$\frac{10}{18} = 0.56$
$0 < x \leq 60$	$\frac{13}{18} = 0.72$
$0 < x \leq 70$	$\frac{18}{10} = 1$

For example:
The 0.11 tells us that 11% of people scored below 10%
The 0.56 tells us that 56% of people scored below 60%
The 0.72 tells us that 72% of people scored below 60%
The 1 tells us that 100% of people scored below 70%

How To Draw A Cumulative Frequency Curve

We plot the upper boundary on the x axis and cumulative frequency (or the relative cumulative frequency) on the y axis.
Connect the points to product a smooth curve, do **not use a ruler** to produce straight lines!



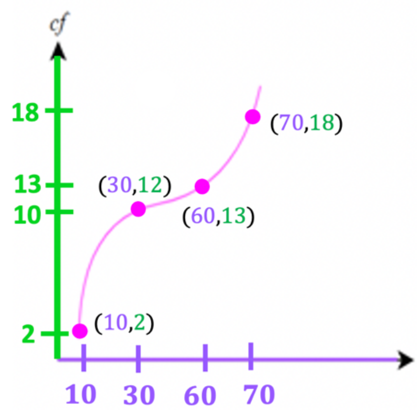
For example:

x	Frequency (f)
$0 < x \leq 10$	2
$10 < x \leq 30$	8
$30 < x \leq 60$	3
$60 < x \leq 70$	5

becomes $\Longrightarrow$

Frequency (f)	Cumulative Frequency (cf)	Hence we plot the point
2	$\xrightarrow{\text{copy}}$ 2	$(10,2)$
8	10	$(30,12)$
3	13	$(60,13)$
5	18	$(70,18)$

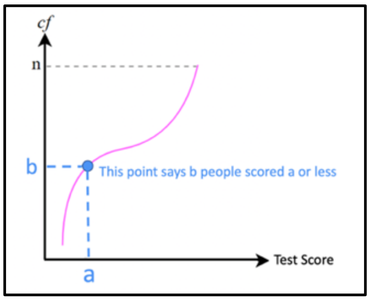
gives the graph $\Longrightarrow$

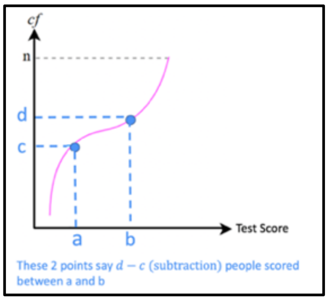


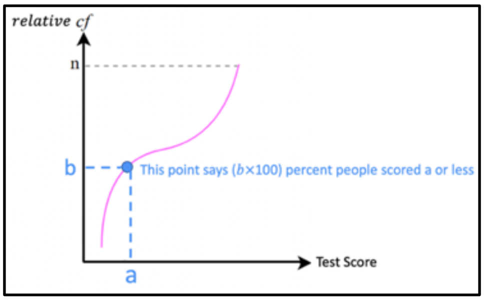
The highest y value on the graph is the total number which is 18

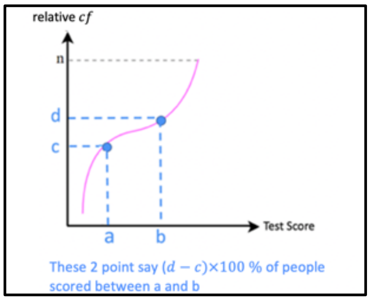
Don't worry if the next 3 sections below do not make sense. They are just a brief summary. There are 2 examples after which makes everything very clear (you may wish to go straight to this section as doing the examples is where the understanding of how to do cumulative frequency comes from). It is a very easy topic!

Interpreting A Cumulative Frequency Graph







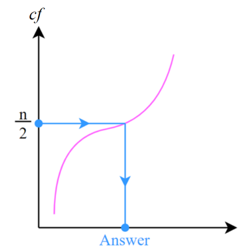


These talk about percentages since the relative frequency is on the y axis

Performing Quartile Calculations – Median, Upper and Lower Quartile

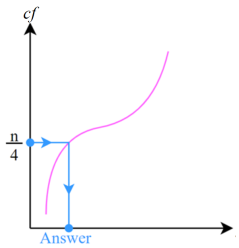
Median:

Locate $\frac{n}{2}$ on the y axis and go across to the curve and down to find the corresponding value on the x axis



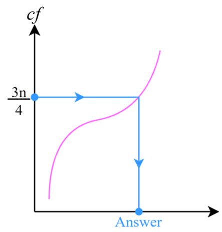
Lower Quartile:

Locate $\frac{n}{4}$ on the y axis and go across to the curve and down to find the corresponding value on the x axis



Upper Quartile:

Locate $\frac{3n}{4}$ on the y axis and go across to the curve and down to find the corresponding value on the x axis



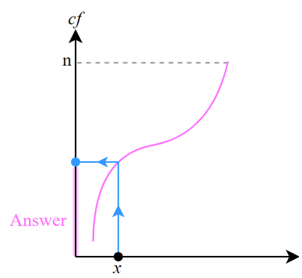
Remember: We always use $\frac{n}{2}, \frac{n}{4}$ or $\frac{3n}{4}$ NEVER $\frac{n+1}{2}, \frac{n+1}{4}$ or $\frac{3(n+1)}{4}$ for the quartiles!!!

Don't make the common mistake of thinking $\frac{n}{2}, \frac{n}{4}$ or $\frac{3n}{4}$ are not your answers, you need to go and find the corresponding value on the x axis!

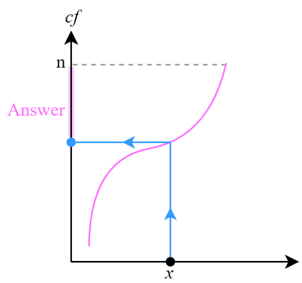
Performing Quartile Calculations – How Many?

Locate the value on the x axis and go up to the curve and across to find the corresponding value on the y axis (since the y axis tells us how many)

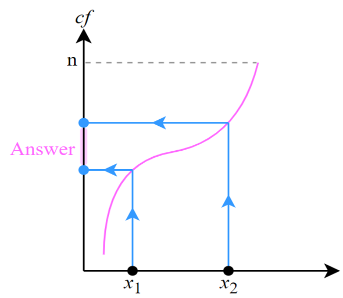
Less than x



More than x



Between 2 given x values

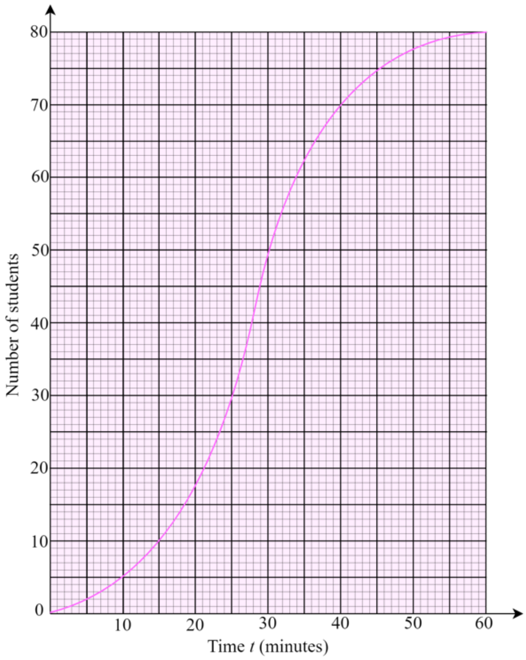


If want any answers as a percentage: $\frac{answer}{n} \times 100$

Examples

Example 1:

1) The following is a cumulative frequency diagram for the time t, in minutes, taken by 80 students to complete a task



Time (minutes)	Number of students
$0 \le t < 10$	
$10 \le t < 15$	
$15 \le t < 30$	
$30 \le t < 40$	
$40 \le t < 60$	

i. Find an estimate for the median
ii. Find an estimate for the lower quartile
iii. Find an estimate for the upper quartile
iv. Find the number of students who completed the task in **less than** 30 minutes
v. Find the number of students who completed the task in **more than** 40 minutes
vi. Find the number of students who took **between** 25 and 35 minutes to complete the task
vii. Given that 45 students took less than k minutes to complete the task, find the value of k.
viii. Complete the frequency table

First you must know that

- Median $= \frac{total\ number}{2} = \frac{n}{2}$
- Lower Quartile $= \frac{total\ number}{4} = \frac{n}{4}$
- Upper Quartile$= \frac{3 \times total\ number}{3} = \frac{3n}{4}$

i.

$\text{Median} = \frac{n}{2} = \frac{80}{2} = 40$

Go to the graph on the next page

Locate the 40th value **on the y axis**
(y axis since that is the number)

Find the corresponding x
(go across and down)

$\text{median} = 28$ minutes

ii.

$\text{LQ} = \frac{n}{4}\frac{80}{4} = 20$

Go to the graph on the next page

Locate the 15th value **on the y axis**
(y axis since that is the number)

Find the corresponding x
(go across and down)

$\text{LQ} = 21$ minutes

iii.

$\text{UQ} = \frac{3n}{4} = \frac{3(80)}{4} = 60$

Go to the graph on the next page

Locate the 45th value **on the y axis**
(y axis since that is the number)

Find the corresponding x
(go across and down)

$\text{UQ} = 34$ minutes

iv.

$\text{IQR} = \text{UQ} - \text{LQ}$

$= 34 - 21$

$= 13$

v.

Go to the graph on the next page

Locate 30 **on the x axis**
(x axis since that is the time)

Find the corresponding y
(go up and across)

50 students

vi.

Go to the graph on the next page

Locate 40 **on the x axis**
(x axis since that is the time)

Find the corresponding y
(go up and across)

70 students

This means less than 40 mins though.
We want **more than** 40 mins, so we subtract from the total
$80 - 70 = 10$ students

vii.

Go to the graph on the next page

Locate 25 and 35 **on the x axis**

Find the corresponding $y's$

$62 - 30 = 32$ students

viii.

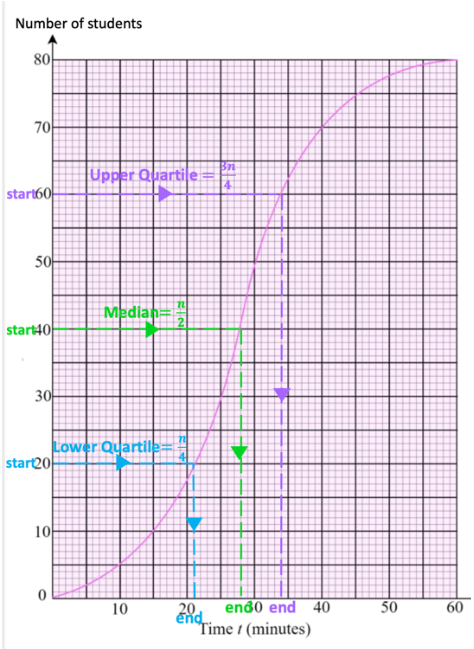
Go to the graph on the next page

Locate 45 **on the y axis**

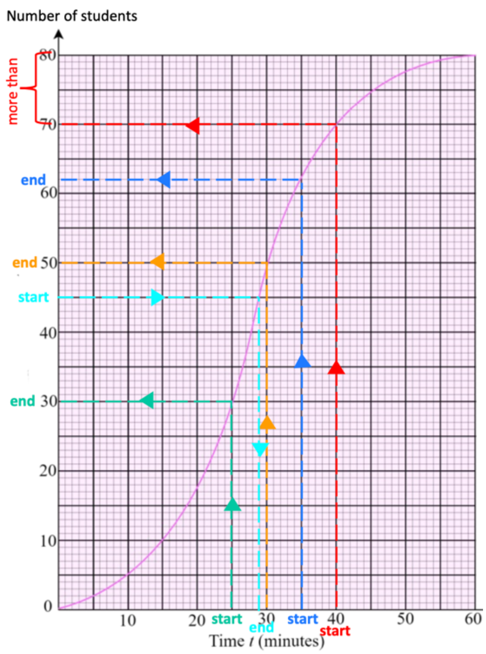
Find the corresponding x

$k = 29$ mins

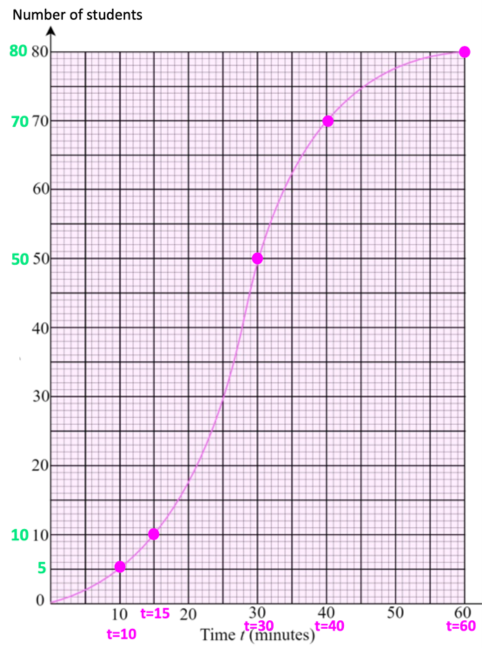
Graph for questions i.-iv.



Graph for questions v.-vii.



Graph for question viii.



viii.

Locate the times on the x axis using the upper boundaries and find the corresponding cumulative frequencies

Time (minutes)	cf
$0 \le t < 10$	5
$10 \le t < 15$	10
$15 \le t < 30$	50
$30 \le t < 40$	70
$40 \le t < 60$	80

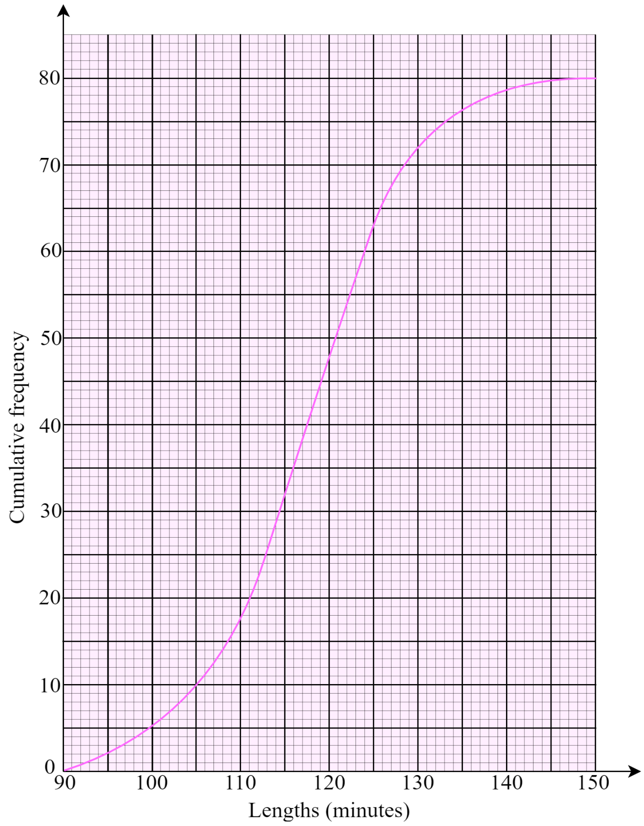
build the frequency column $\Longrightarrow$

Time (minutes)	Number of Students
$0 \le t < 10$	5
$10 \le t < 15$	$10 - 5 =$
$15 \le t < 30$	$50 - 10 = 40$
$30 \le t < 40$	$70 - 50 = 20$
$40 \le t < 60$	$80 - 70 = 10$

simplify the numbers $\Longrightarrow$

Time (minutes)	Number of Students
$0 \le t < 10$	5
$10 \le t < 15$	5
$15 \le t < 30$	40
$30 \le t < 40$	20
$40 \le t < 60$	10

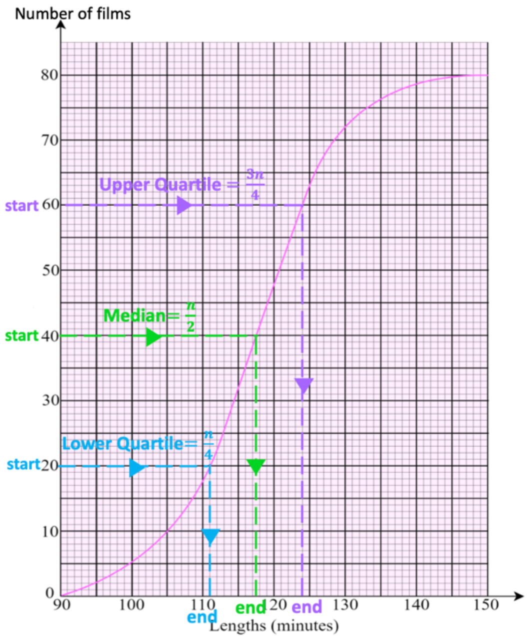
Example 2: The cumulative frequency graph shows information about the length, in minutes of 80 films



i. How many films are there in total?
ii. Find an estimate for the median
iii. Find an estimate for the interquartile range
iv. How many films lasted **less than** 130 mins
v. How many films **more than** 120 mins
vi. Find an estimate for the **percentage** of the 80 films that lasted **more than** 125 minutes
vii. Find the number of films who took **between** 100 and 110 minutes to complete the task
viii. Given that 40 students took less than k minutes to complete the task, find the value of k.

i. 80 films	ii. $\text{Median} = \frac{80}{2} = 40$ Locate the 40th value **on the y axis** Find the corresponding x $\text{median} = 117.5$ minutes	iii $\text{LQ} = \frac{80}{4} = 20$ Locate the 20th value on the y axis Find the corresponding x $\text{LQ} = 111$ $\text{UQ} = \frac{3(80)}{4} = 60$ Locate the 60th value on the y axis Find the corresponding x $\text{UQ} = 124$ $\text{IQR} = \text{UQ} - \text{LQ}$ $= 124 - 111$ $=$ 13 minutes	iv. Locate 130 **on the x axis** Find the corresponding y 72 films
v. Locate 120 **on the x axis** Find the corresponding y 48 films This means less than 120 mins though. We want **more than** 120 mins so we subtract from the total $80 - 48 = 32$ films	vi. Locate 125 on the x axis Find the corresponding y 63 films $\frac{80-63}{80} = \frac{17}{80} \times 100 = 21.2\%$ of films	vii. Locate 100 and 110 **on the x axis** Find the corresponding y's $18 - 5 = 13$ films	viii. Locate 40 **on the y axis** Find the corresponding x $k = 123$ mins

Graph for questions i.-iii.



Graph for questions iv.-viii.